

**FOCUSING PROPERTIES OF A THICK RECTANGULAR METAL GRID OF FINITE SIZE.  
PLANAR GRID**

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## **ABSTRACT**

The MoM approach in frequency domain with the direct factorization of the symmetric impedance matrix is an appropriate method for the analysis of the near zone and Fresnel zone of the finite metal grids. These grids are of interest for focusing purposes. When the number of unknowns is on the order of 10,000, the grids on the size of 6 wavelengths may be modeled, with a reasonable degree of accuracy and over a wide range of frequencies.

It has been found that a planar metal grid of finite size has a well-developed broadband focusing zone, which occurs at relatively small (up to 15-20 degrees) axial incidence angles and for arbitrary polarization of the incident wave. The grid size is on the order of several wavelengths. The number of rectangular cells is 36-100. The critical parameters are the finite size of the grid and its large thickness (depth). The focal magnitude increases with increasing the depth of the grid and the number of cells of the finite grid. The off-axis position of the focal zone is very sensitive to the incidence angle. The frequency bandwidth of the focal zone may be as high as 40%. Refs. 14.

### Keywords:

**metal grid, thick metal grid, metal lens antenna, microwave focusing, Fresnel zone focusing, MoM method.**

## I. INTRODUCTION

One basic type of metal lens antennas is a thick curved metal grid of variable thickness [1]. Every grid cell is a waveguide where the phase speed of EM waves is higher than in free space. Use of waveguides of different lengths causes a proper phase shift at different distances from the axis. This enables focusing of the primarily plane wave at a point (or zone) of interest. Due to higher phase speed we should use a concave metal lens instead of convex to achieve focusing [1].

Yet another type is a Fresnel zone plate or a zone plate antenna [2, 3]. The focusing is achieved by proper phase correction in one plane, according to Fresnel zones at a given focal distance and frequency. The simplest (small-angle) zone plate is semi-opaque, with every other (out of phase) Fresnel zone blocked out by metal rings. The zone plates [4-6] are more practical than the thick curved metal grids.

The present paper examines focusing properties of a regular (rectangular) metal grid of finite size. An inviting property of this grid is its easy fabrication. Another inviting property is the near same behavior of the grid for different polarizations of the incident signal.

If the grid would be infinitely thin, it could be modeled as a finite number of rectangular apertures. Every rectangular aperture is a weakly focusing system by itself [7]. However, the entire grid is hardly expected to have noticeable focusing properties, mostly due to phase mismatch between different apertures. On the other hand, the grid of any thickness but infinite extend also doesn't have any focusing properties, unless a defect is introduced somewhere [8]. Therefore, one may expect that a finite metal grid of any thickness does not have noticeable focusing properties.

Surprisingly, the numerical analysis indicates existence of a rather sharp focal zone for certain finite thick grids, with the intensity gain of 7 dB and higher. Two critical components are the finite size of the grid and its finite thickness (depth). The finite grid curvature is not required. The entire grid is thus flat.

The MoM simulation tool is chosen as the method of the analysis. The near zone and the Fresnel zone of the finite grid are accurately described using that method, at a large number of frequencies of interest, for the grids whose sizes are less than 6-7 wavelengths.

## II. MoM METHOD

### 2.1. Filling the symmetric impedance matrix

The MoM formulation is rather standard and utilizes the mixed-potential approach to EFIE and the RWG basis functions [9,10]. We use the symmetric impedance matrix, which allows for a faster speed and larger memory storage. For a set of basis functions  $\vec{f}_m^S(\vec{r})$ ,  $m = 1 \dots N$  covering the entire scattering surface  $S$  (the test functions are the same) the impedance matrix of the size  $N \times N$  is derived to the form

$$Z_{mn} = \frac{j\omega\mu_0}{4p} \iint_{S_m S_n} \vec{f}_m^S(\vec{r}) \cdot \vec{f}_n^S(\vec{r}') g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} - \frac{j}{4p\omega\epsilon_0} \iint_{S_m S_n} (\nabla_s \cdot \vec{f}_m^S(\vec{r})) (\nabla_s \cdot \vec{f}_n^S(\vec{r}')) g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} \quad (1)$$

where  $g = \exp(-jkR)/R$ ,  $R = |\vec{r} - \vec{r}'|$  is the free-space Green's function (time dependency  $\exp(j\omega t)$  is assumed everywhere). Utilizing the RWG basis functions [10], every integral in the first term on the right-hand-side of equation (1) is represented through a term

$$A_{pq}^{ij} = \iint_{t_p t_q} (\vec{r}_i \cdot \vec{r}_j') g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} \quad p, q = 1, \dots, P \quad (2)$$

to within a constant factor. Here,  $\vec{r}_i = \vec{r} - \vec{r}_i$  for any vertex  $i$  of triangular patch  $p$  whereas

$\vec{r}_j' = \vec{r}' - \vec{r}_j$  for any vertex  $j$  of patch  $q$ . Total number of patches is  $P$ . Similarly, every integral

in the second term on the right-hand-side of equation (1) is built based upon a term

$$\Phi_{pq} = \iint_{t_p t_q} g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} \quad p, q = 1, \dots, P \quad (3)$$

Note that, due to symmetry,  $\Phi_{pq} = \Phi_{qp}$ ,  $\bar{A}_{pq}^{ij} = \bar{A}_{qp}^{ji}$ . The double surface integrals are calculated using the Gaussian formulas of the *same* (4<sup>th</sup> or 1<sup>st</sup>) degree of accuracy [11]. The lowest degree of accuracy is used when the distance between two patches is 15 times greater than their maximum size.

The singular portions of self-integrals ( $p = q$  in equations (2), (3)) are found analytically, using the closed-form solutions given in [12].

Further, the contributions of every pair of patches  $p, q$  given by equations (2) and (3) are accumulated in the appropriate terms of equation (1). This happens when the basis functions  $m$  and  $n$  have  $p$  and  $q$  either as plus or minus triangles. The summation process for the impedance matrix follows

$$Z_{mn} = Z_{mn} \pm \frac{l_m l_n}{A_p A_q} \left\{ \frac{j\omega m_0}{16p} A_{pq}^{ij} - \frac{j}{4pwe_0} \Phi_{pq} \right\} \quad (4)$$

Here,  $m = m(p)$  are numbers of those RWG basic functions, which include patch  $p$ . Similarly, indexes  $n = n(q)$  indicate basis functions, which include patch  $q$ .  $l_m, l_n$  are the base edges. Plus sign occurs when  $p$  and  $q$  are both “plus” or “minus” triangles. Otherwise, sign minus should be chosen. Index  $i = i(m)$  corresponds to the free vertex of patch  $p$  in element  $m$ . Index  $j = j(n)$  corresponds to the free vertex of patch  $q$  in element  $n$ .

## 2.2. Near-field

Using two potentials for charges and currents on the metal surface, the scattered fields  $\vec{E}^s$  and  $\vec{H}^s$  are written in terms of the solution of MOM equations,  $I_n$ , in the form

$$\begin{aligned} \vec{E}^s &= \sum_{n=1}^N \left\{ \frac{k^2}{j\omega\epsilon_0} \int_{S_n} \vec{f}_n^s(\vec{r}') g(\vec{r}, \vec{r}') d\vec{r}' + \frac{1}{j\omega\epsilon_0} \int_S (\nabla_{\vec{r}'} \cdot \vec{f}_n^s(\vec{r}')) \nabla_{\vec{r}} g(\vec{r}, \vec{r}') d\vec{r}' \right\} I_n \quad \vec{r}' \text{ on } S \\ \vec{H}^s &= \sum_{n=1}^N \left\{ \int_S \nabla_{\vec{r}} \times (g(\vec{r}, \vec{r}') \vec{f}_n^s(\vec{r}')) d\vec{r}' \right\} I_n \quad \vec{r}' \text{ on } S \end{aligned} \quad (5)$$

Similar to the approach to the impedance matrix, all integrals in equations (5) are formed out of three base integrals over triangular patches

$$\bar{\mathbf{K}}_p^i(\vec{r}) = \int_{t_p} \vec{r}_i g(\vec{r}, \vec{r}') d\vec{r}' \quad p = 1, \dots, P \quad (6)$$

$$\bar{\mathbf{L}}_p(\vec{r}) = \int_{t_p} \nabla_r g(\vec{r}, \vec{r}') d\vec{r}' \quad p = 1, \dots, P \quad (7)$$

$$\bar{\mathbf{M}}_p^i(\vec{r}) = \int_{t_p} \vec{r}_i \cdot (\nabla_{\vec{r}} \times g(\vec{r}, \vec{r}')) d\vec{r}' \quad p = 1, \dots, P \quad (8)$$

to within a constant factor. Here,  $\vec{r}_i = \vec{r} - \vec{r}_i$  for any vertex  $i$  of patch  $p$ . The integrals (6-8) are found using Gaussian integration of 4<sup>th</sup> order when the distance to a metal patch is smaller than its maximum size times 15. Alternatively, the central-point approximation is used.

This approach leads to reasonably accurate results for the near field when the distance from the closest triangular patch to the observation point is greater than the size of that patch.

### 2.3. Computations

Factorization of the symmetric impedance matrix is done using the double precision simple driver ZSYSV of the LAPACK library [13] for symmetric complex matrixes. It is linked against the library libmwlapack.dll in the Matlab® environment. The rest of the codes are written in C++ and then converted to Matlab® dynamically linked libraries using Intel v.7 C++ compiler. The entire code runs in the Matlab® environment. The code allows for maximum 11,500 unknowns and has the speed of about 20 min per frequency step for a structure with 10,992 unknowns on a PIV 2.4 GHz processor (LINUX). The code and its preliminary description are available from [14] for both Windows XP and UNIX platforms (Matlab® version 6.5 is required).

### III. TEST

The performance of the solver was tested by comparison with the Ansoft HFSS simulations in the near field for two different structures: a square plate (Fig. 1a) and a simplest metal grid (Fig. 2a). Fig. 1a shows the first structure, a plate of the size  $a$  by  $a$ . A z-polarized wave is incident on the structure along the positive x-axis as shown in the figure. Fig. 1b shows the plot of the magnitude of the electric field as a function of the axial distance on the plate axis. The results are compared with the solution obtained using Ansoft HFSS v 8.5. The number of unknowns in the case of Ansoft HFSS is 65,445. The number of unknowns for the plate (24x24 rectangles) is 1,680. Fig. 1c shows the plot of the magnitude of the magnetic field as a function of axial distance for the same configuration. The values of the electric and magnetic fields are calculated at axial distances less than or equal to 15 cm from the plate.

The MoM solver accurately reproduces the near field, except for the magnetic field in the immediate vicinity of the plate, when the observation distance from the plate is smaller than or equal to the size of a mesh patch. This circumstance was explained in the previous section. In order to check the solver convergence, Table 1 presents the  $L_2$ -relative error of the near field simulation for different MoM meshes. The same axial data for two fields was used as shown in Fig. 1. It is interesting to note that the minimum  $E$ -field error is achieved for the mesh with 24x24 rectangles. Then, the error increases. This perhaps indicates the insufficient meshing accuracy of the corresponding ANSOFT HFSS solution. The  $H$ -field error continuously reduces. This is mostly due to a smaller thickness of the central gap shown in Fig. 1c.

Similar simulations were carried out for a grid structure shown in Fig. 2a. The entire grid has dimensions of  $a$  by  $a/5$ . A z-polarized wave is incident on the structure along the positive x-axis and the near field is again calculated. Similar to the previous case, the values of the electric and magnetic fields are calculated at axial distances less than or equal to 15 cm from the grid. Fig. 2b shows the magnitude of the axial electric field as a function of the axial distance. The corresponding ANSOFT HFSS solution is shown by a dotted line. Fig 2c shows the magnitude of

the magnetic field as a function of axial distance. The corresponding ANSOFT HFSS solution is again shown by a dotted line. The size of tetrahedral mesh in the case of Ansoft HFSS is 82,524. The number of unknowns for the grid (12x6 rectangles per cell side) is 1,632.

Figs. 2b and c demonstrate a reasonable accuracy in the reproduction of the near field at a relatively small number of unknowns employed by the MoM solution. To complete the results, Table 2 gives the convergence history for the grid structure, similar to Table 1 for the plate. The error continuously decreases when the size of the mesh increases. Better results are observed when the mesh size in the dominant direction of the surface current (the z-direction) is made finer. Similar observation also holds for the plate in Fig. 1.

## IV. RESULTS

### 4.1. Frequency behavior of the on-axis Poynting vector

The near field of metal grids of different thicknesses is characterized in terms of the magnitude of the Poynting vector at different points in space and at different frequencies. The MoM structure mesh is always created in such a way that there are at least 8 triangular patches per wavelength at a highest frequency. The test results indicate that the local/integral error in the near field is not expected to exceed 15% at the highest frequency in that case. The error at lower frequencies is considerably smaller.

In order to check the frequency behavior, we first calculate the total field only on the grid axis at different frequencies. The resulting data is arranged in a 3D plot where the  $x$ -axis is the ratio grid size/wavelength. The total size of the square grid  $s$  is fixed at 10 cm. The frequency sweep covers the range  $0.2 < \frac{s}{\lambda} < 6$ . In all the figures presented in this subsection, the  $y$ -axis of the half-tone plot is the axial distance from center of the grid, which changes from -20 cm to +20cm (from  $-2s$  to  $2s$ ). Negative distances correspond to illuminated zone, whereas positive distances are in the shadow zone. Any  $x$ -cut of the plot at a given frequency/wavelength provides with the on-axis behavior of the total Poynting vector (magnitude of the Poynting vector of the

total field) as a function of axial distance. The grey scale extends from white (zero Poynting vector magnitude) to black (maximum Poynting vector magnitude for a given plot). The colorbar on the right shows the relative intensity level. The data is normalized versus the Poynting vector density of the incident field so that the intensity level of 1 always corresponds to the Poynting vector density in the incident signal.

Fig. 3 shows the near field of a square grid with 8 full cells. A z-polarized plane wave with  $E = 1$  V/m is normally incident on it along the positive x-axis. The grid thickness gradually increases as  $s/24$  (Fig. 3a),  $s/12$  (Fig. 3b), and  $s/9.6$  (Fig. 3c). The number of MoM unknowns is 3,936, 8,640, and 10,992, respectively.

The most notable property of the near field is the appearance of a focusing zone (a “black spot” in Figs. 3 behind the planar grid, in the right lower corner of all three figures. The maximum relative magnitude of the Poynting vector in the focus increases, when the grid thickness increases. It reaches 5.17 for the thickest mesh in Fig. 3. Similar results were obtained for other tested grids with 6x6 and 10x10 cells, respectively. If grid thickness is kept the same, the focusing magnitude also increases with increasing the number of the cells of the finite grid. The focusing zone is broadband and has the bandwidth of approximately 40%. Its center frequency depends on both the cell size and the cell depth. It is approximately given by  $l/2 = d$  for thin grids. For thick grids,  $l/2 < d$ .

#### 4.2. Distribution of the near field at the center frequency

Fig. 4 presents the cross-section of the near field (in the xy-plane that is orthogonal to the grid and includes the origin) at the “center” frequency of the focusing zone. The grid in Fig. 3c is considered with  $s/l = 5.2$ . Fig. 4a corresponds to normal incidence. It is seen that the focusing zone in Fig. 4a is localized exactly on the axis of the grid.

Fig. 4b gives the same results but for the incidence angle of 5 degrees versus the x-axis. The polarization (along the z-axis) remains the same. The focus is shifted to approximately 15 degrees from the x-axis. When the incidence angle changes to 10 degrees (Fig. 4c), the focus is

already shifted to approximately 30 degrees from the axis. The focal zone thus moves linearly with the incidence angle, at least at small incidence angles. The response angle is approximately three times the incidence. The sign of the response angle in the  $xy$ -plane is the same.

In order to check if these results are a common feature we changed polarization to 45 degrees versus the  $z$ -axis in the  $yz$ -plane and repeated the simulations of Fig. 4. Fig. 5 shows the corresponding data for the near field. Compared to Fig. 4, the focusing mechanism remains nearly unchanged. The magnitude of the Poynting vector at the focus is only a little smaller. Similar results were obtained for other tested grids with  $6 \times 6$  and  $10 \times 10$  cells, respectively. Further change in polarization does not alter the results since the mesh is symmetric with regard to 90-degree rotation.

#### 4.3. Higher angles of incidence

In order for the focusing mechanism to work, the normal incidence angle must be smaller than 15 degrees. Fig. 6 shows the data similar to Fig. 3 but for higher angles of incidence (15 and 30 degrees versus the  $x$ -axis in the  $xy$ -plane, respectively). The polarization is along the  $z$ -axis. The on-axis near field as a function of frequency is again plotted. Figs. 6a, b correspond to incidence angles of 15 and 30 degrees for the thin grid shown in Fig. 3a. Figs. 6c, d correspond to incidence angles of 15 and 30 degrees for the grid shown in Fig. 3b. Figs. 6e, f correspond to incidence angles of 15 and 30 degrees for the thick grid shown in Fig. 3c, respectively.

One can see that the focusing mechanism is nearly lost, at least in the visible range of frequencies. The black spots on the right-lower corner of the plots have considerably smaller magnitudes and are not well localized. It seems that they might have been shifted toward higher frequencies and greater distances from the mesh.

#### 4.4. Mesh versus solid plate

Finally, it is interesting to compare the near field of the grid with the near field of the solid plate of the same size. Fig. 7 presents the on-axis Poynting vector for the plate as a function of frequency. The number of MoM unknowns for the plate ( $48 \times 48$  rectangles) is 6,816. Three

angles of incidence are 0 (Fig. 7a), 15 (Fig 7b) and 30 (Fig. 7c) degrees versus the x-axis in the xy-plane, respectively. The polarization is along the z-axis.

Compared to the mesh, the plate provides nearly perfect isolation of the backscattering near field at higher frequencies (when  $s/l > 3$ ). However, the isolation at lower frequencies may be somewhat better for a thick grid.

## V. CONCLUSIONS

It has been found that a planar metal grid of finite size has a well-developed broadband focusing zone, which occurs at relatively small (up to 15-20 degrees) axial incidence angles and for any polarization of the incident wave. The grid size is on the order of several wavelengths. The number of rectangular cells is 36-100. The critical parameters are the finite size of the grid and its finite thickness (depth). The focal magnitude increases with increasing the depth of the grid and the number of cells of the finite grid. The off-axis position of the focal zone is very sensitive to the incidence angle. The focusing zone bandwidth may reach 40%.

The MoM tool in the frequency domain with the direct factorization of the symmetric impedance matrix is an appropriate method for the analysis of the near zone and Fresnel zone of the finite metal grids. When the number of unknowns is on the order of 10,000-11,000, the grids on the size of 6 wavelengths may be modeled, with a reasonable degree of accuracy. Frequency sweeps with the total of 200 discrete points are performed in a reasonable amount of time. It was estimated that the ANSOFT HFSS solutions, having the same mesh accuracy on the grid, would require about 500% more execution time to perform. Grids with a larger number of cells and a higher depth should be considered, at higher size/wavelength ratios. A reliable sub-block iterative method for the solution of MoM equations should be employed in that case.

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## FIGURE CAPTIONS

Fig.1. a) – Square plate (1,680 MoM unknowns); b) - magnitude of the electric field as a function of axial distance; c) - magnitude of the magnetic field as a function of axial distance.

Fig.2. a) – Simplest metal grid (1,632 MoM unknowns); b) - magnitude of the electric field as a function of axial distance; c) - magnitude of the magnetic field as a function of axial distance.

Fig.3. Near field of three metal grids of different thickness. The  $z$ -polarized wave is incident along the positive  $x$ -axis and the magnitude of the near field (Poynting vector) along the grid axis is computed as a function of frequency – right. a) - Grid with the depth of  $s/24$  (3,936 MoM unknowns); b) - grid with the depth of  $s/12$  (8,640 MoM unknowns); c) - grid with the depth of  $s/9.6$  (10,992 MoM unknowns).

Fig. 4. Cross-section of the near field (in the  $xy$ -plane that is orthogonal to the grid and includes the origin) at the “center” frequency of the focusing zone  $s/l = 5.2$  when a  $z$ -polarized plane wave is incident on the grid shown in Fig.3c. a) - normal incidence; b) - angle of incidence is 5 degrees versus the  $x$ -axis; c) - angle of incidence is 10 degrees versus the  $x$ -axis.

Fig. 5. Cross-section of the near field (in the  $xy$ -plane that is orthogonal to the grid and includes the origin) at the “center” frequency of the focusing zone  $s/l = 5.2$  when a 45-degree polarized wave is incident on the grid shown in Fig.3c; a) - normal incidence; b) - angle of incidence is 5 degrees versus the  $x$ -axis; c) - angle of incidence is 10 degrees versus the  $x$ -axis.

Fig.6. a-b) – Magnitude of the near field as a function of frequency computed for the mesh in Fig. 3a with a  $z$ -polarized wave incident at the angle  $15^\circ$  and  $30^\circ$  with respect to the  $x$ -axis; c-d) –the

same data computed for the mesh shown in Fig. 3b; e-f) – the same data computed for the mesh shown in Fig. 3c.

Fig.7. Near field of the square solid plate. a) – normal incidence of the z-polarized wave; b) –  $15^\circ$  incidence of the z-polarized wave; c) –  $30^\circ$  incidence of the z-polarized wave.

## TABLES

Table1. Convergence of the MoM near-field solution as a function of the mesh discretization for a plate (Fig. 1). Ansoft HFSS v. 8.5 results are for a 65,445 tetrahedral mesh. The mesh shown in Fig. 1 is marked bold.

| Mesh discretization (full rectangles) | 8 by 8 | 12 by 12 | 18 by 18 | <b>24 by 24</b> | 36 by 36 | 48 by 48 | 60 by 60 |
|---------------------------------------|--------|----------|----------|-----------------|----------|----------|----------|
| Percentage error, $H$ -field          | 10.96  | 8.70     | 6.90     | 5.87            | 4.66     | 3.86     | 3.24     |
| Percentage error, $E$ -field          | 2.47   | 1.83     | 1.35     | 1.24            | 1.26     | 1.28     | 1.30     |

Table2. Convergence of the MOM near-field solution as a function of the mesh discretization for a grid (Fig. 2). Ansoft HFSS v. 8.5 results are for a 82,524 tetrahedral mesh. The grid shown in Fig. 2 is marked bold.

| Mesh discretization (cell length/height) | 4 by 2 | 6 by 3 | 8 by 4 | 10 by 5 | <b>12 by 6</b> | 16 by 8 | 20 by 10 |
|------------------------------------------|--------|--------|--------|---------|----------------|---------|----------|
| Percentage error, $H$ -field             | 13.18  | 8.45   | 6.16   | 5.03    | 4.35           | 3.64    | 3.24     |
| Percentage error, $E$ -field             | 10.39  | 7.11   | 5.59   | 4.64    | 4.21           | 3.66    | 3.31     |

| Mesh discretization (cell length/height) | 4 by 4 | 6 by 6 | 8 by 8 | 10 by 10 | 12 by 12 | 16 by 16 | 20 by 20 |
|------------------------------------------|--------|--------|--------|----------|----------|----------|----------|
| Percentage error, $H$ -field             | 6.24   | 3.92   | 2.84   | 2.37     | 2.17     | 1.99     | 1.92     |
| Percentage error, $E$ -field             | 6.83   | 4.81   | 4.00   | 3.62     | 3.24     | 2.86     | 2.68     |

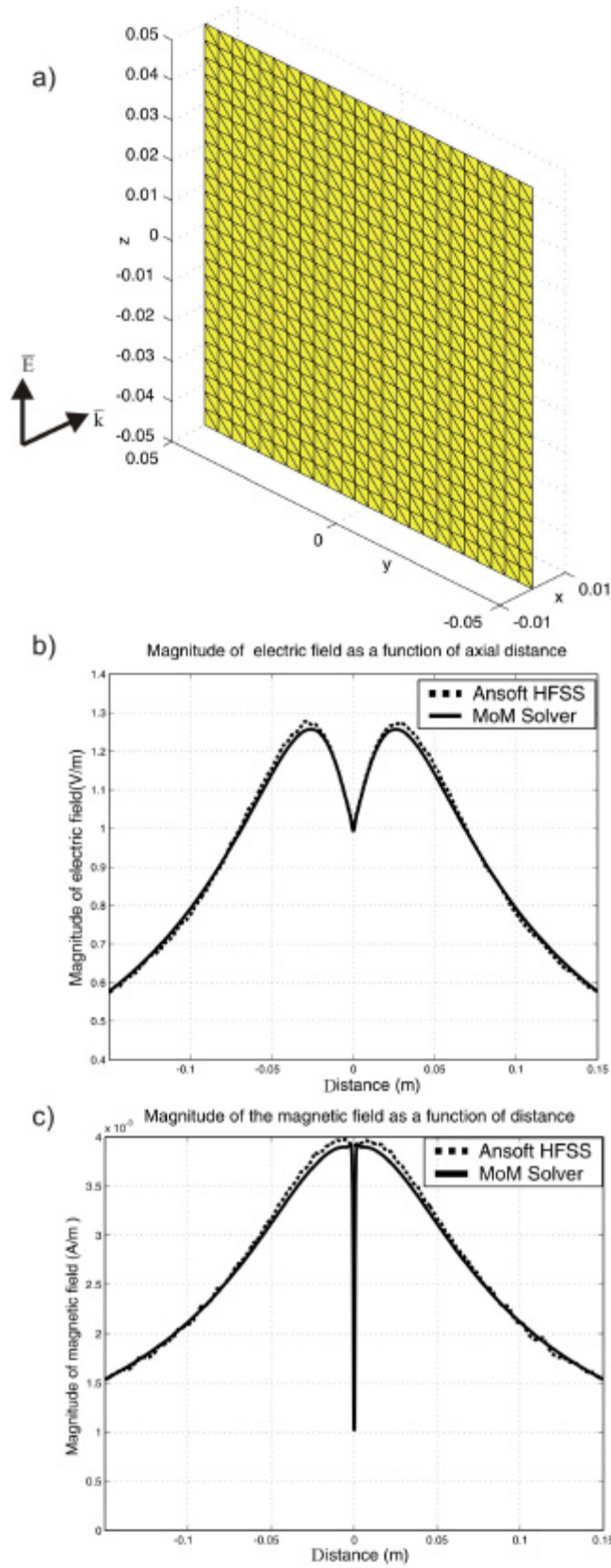


Fig.1

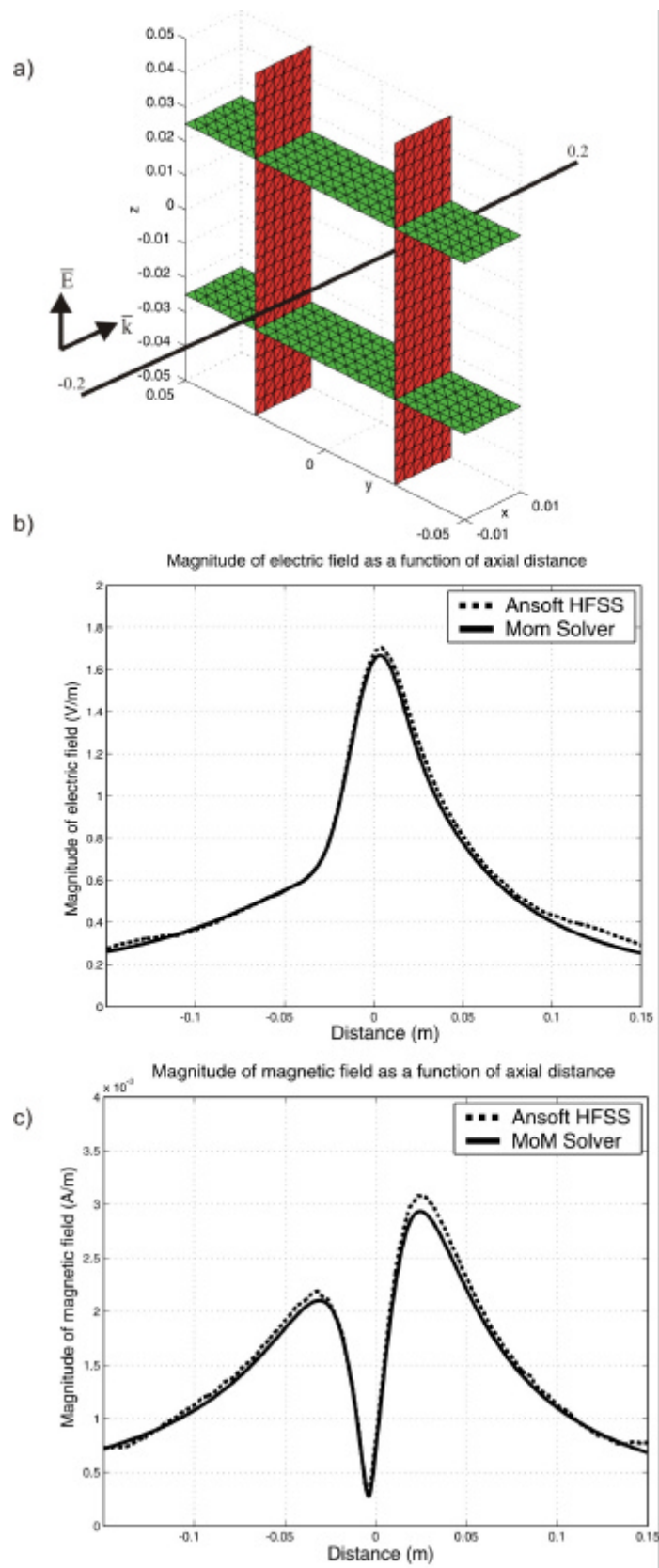


Fig.2

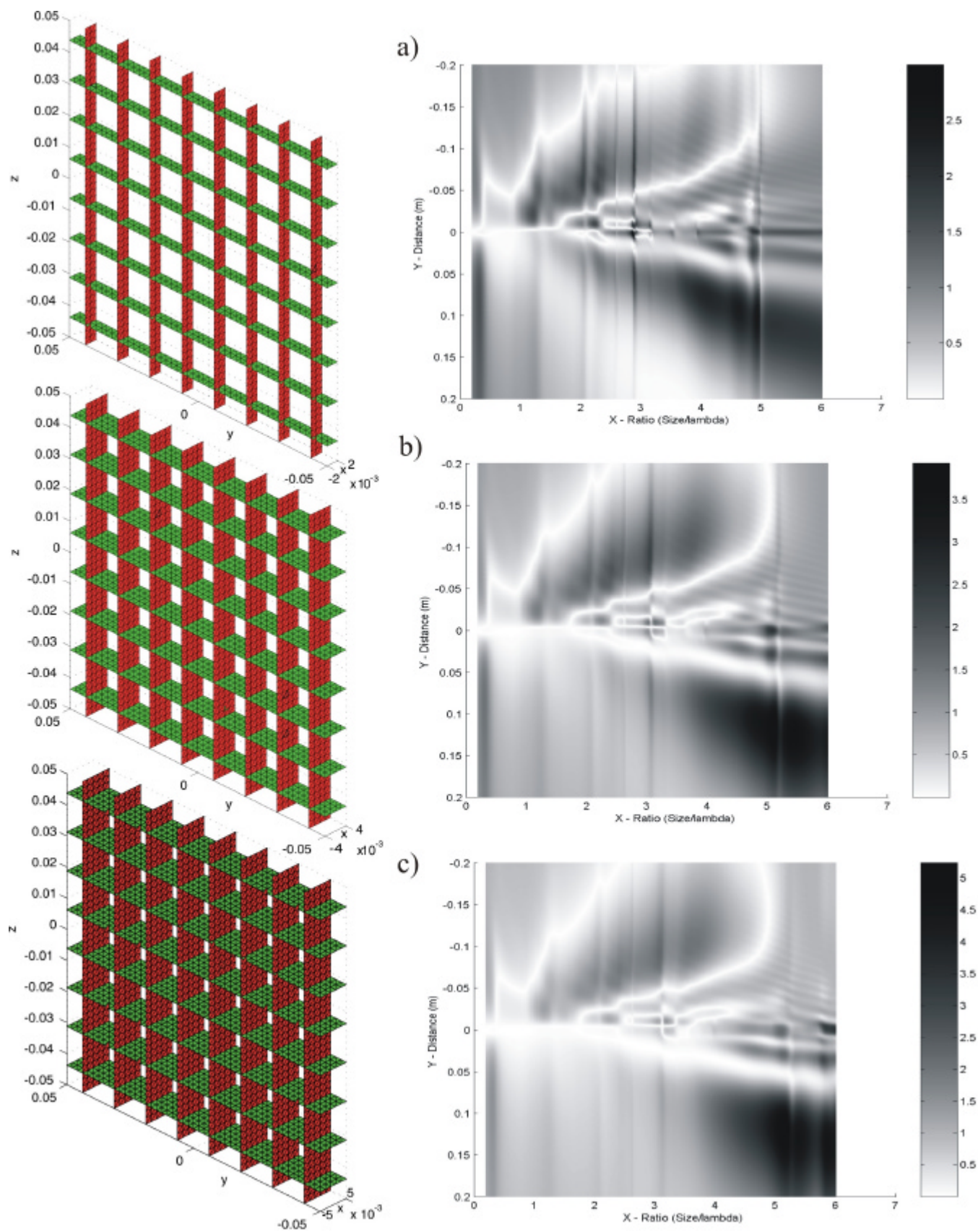


Fig.3

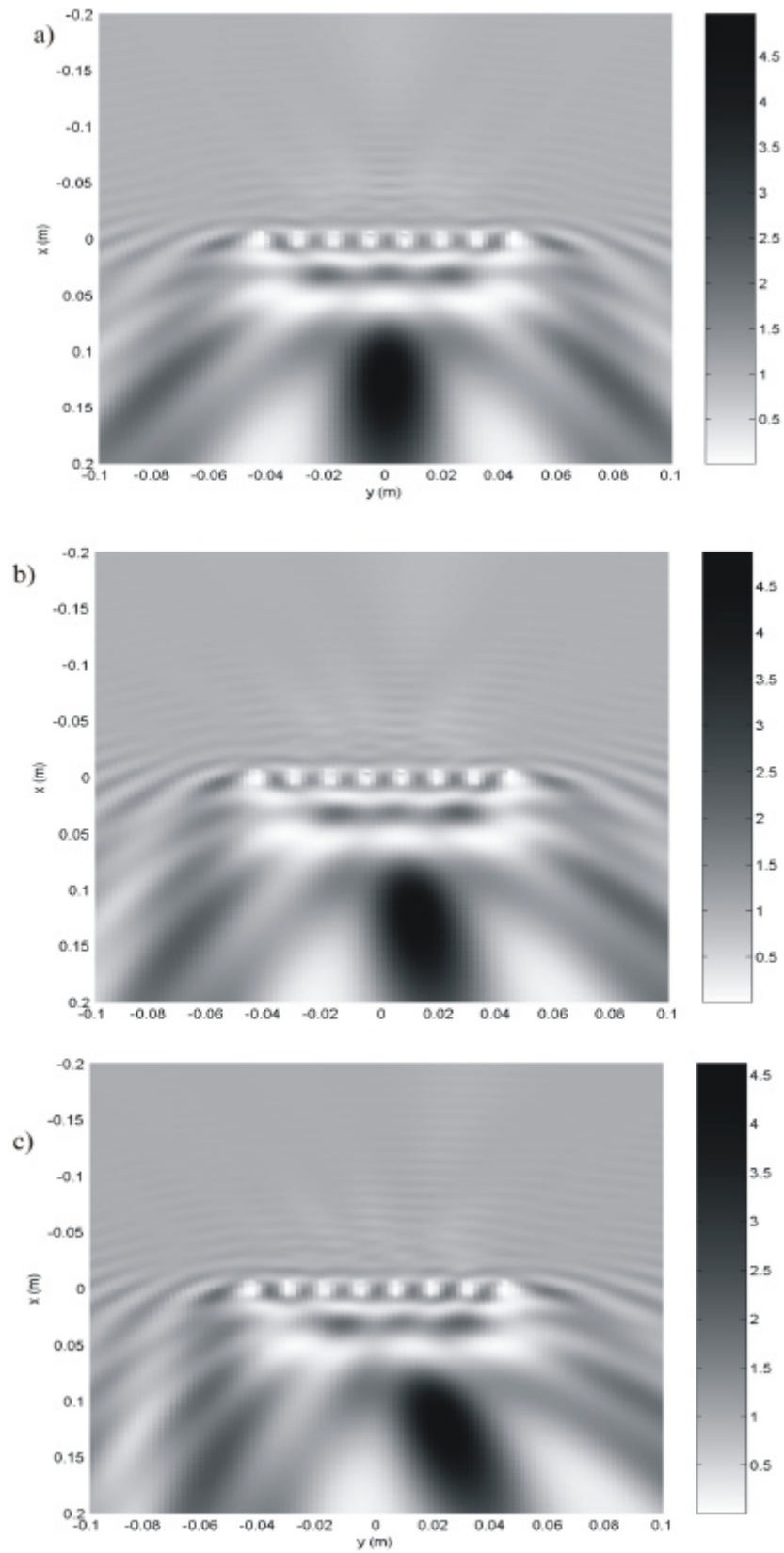


Fig.4

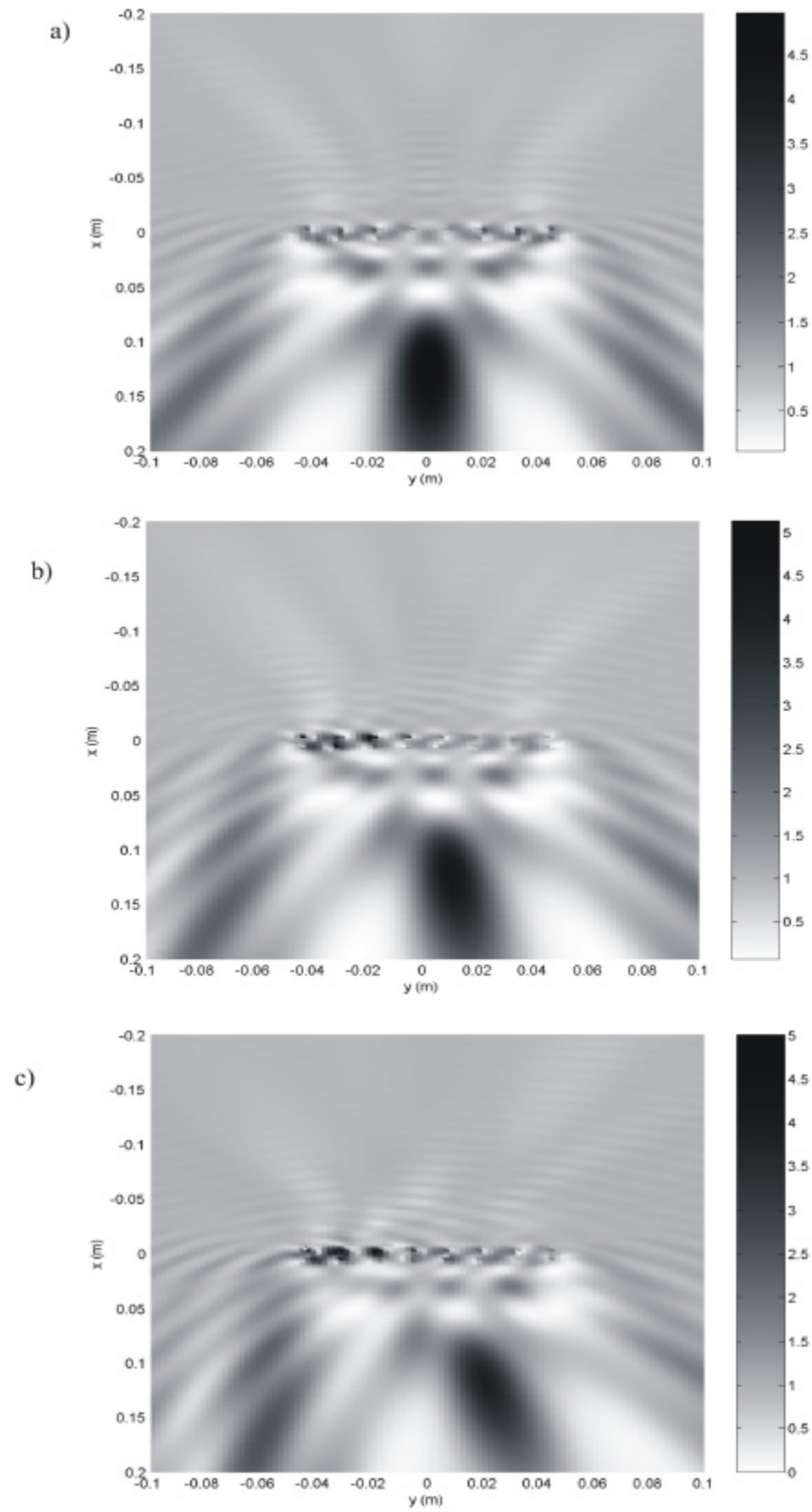


Fig.5

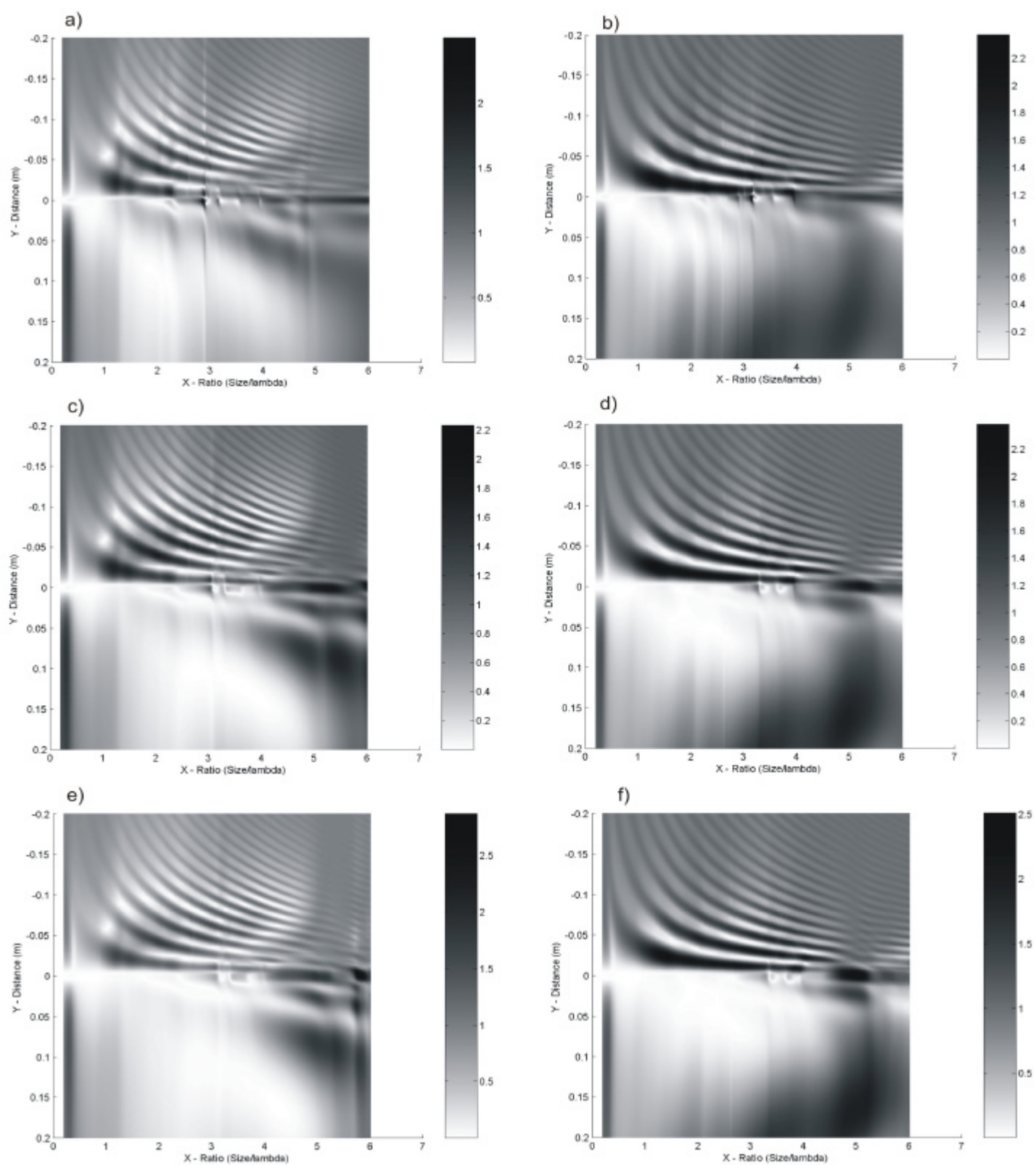


Fig.6

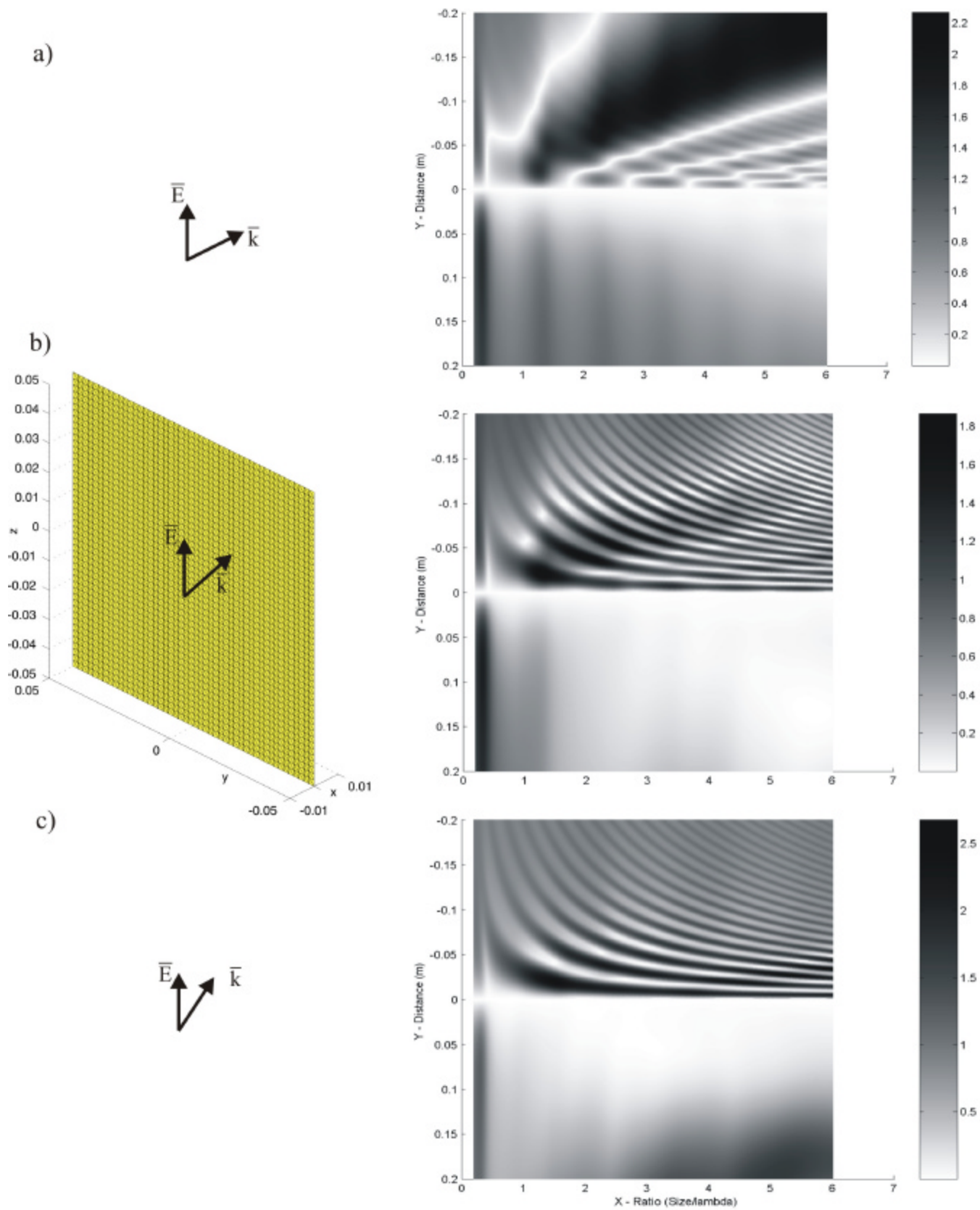


Fig.7