

2. Solver

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2.1 SURFACE RWG BASIS FUNCTIONS

In this section we discuss the surface RWG basis functions used for modeling the metal surface and the Method of Moment solution.

The metal surface is divided into triangular patches as shown in Fig.1

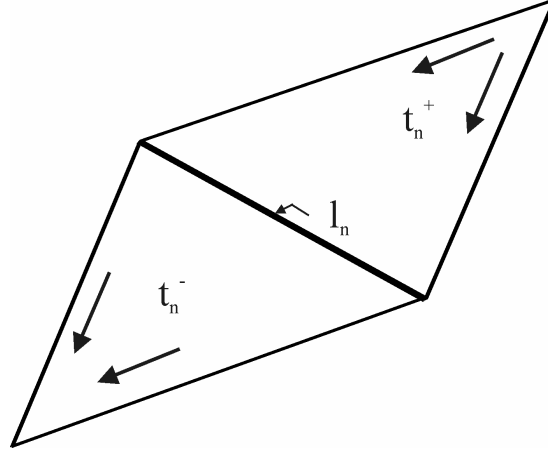


Fig.1 Surface RWG basis function

For any two triangular patches, t_n^+ and t_n^- , having areas A_n^+ and A_n^- , respectively, and sharing the common edge l_n , the n^{th} basis function is defined as

$$\vec{f}_n^s(\vec{r}) = \begin{cases} \frac{l_n}{2A_n^+} \vec{r}_n^+ & \vec{r} \text{ in } t_n^+ \\ \frac{l_n}{2A_n^-} \vec{r}_n^- & \vec{r} \text{ in } t_n^- \end{cases} \quad (1)$$

where $\vec{r}_n^+ = \vec{r} - \vec{r}_n^+$ is the vector drawn from free vertex of triangle t_n^+ to the observation point; $\vec{r}_n^- = \vec{r}_n^- - \vec{r}$ is the vector drawn from the observation point to the free vertex of triangle t_n^- . The basis function is zero outside two adjacent triangles t_n^+ and t_n^- .

2.2 COMPUTE PARAMETERS FOR METAL TRIANGLE

rwgm

Computes parameters for metal triangles including self-integrals.

Syntax:

[geom]= rwgm (P, t, IndexF)

Description:

The function `rwgm` returns a structure `geom` which defines all the parameters of the metal triangles including self integrals. The self integrals for metal are calculated using the functions `analytma` and `analytmb`. The input parameters to the function are `IndexF` which defines the number of integration points and the `P` and `t` matrices which correspond to the vertices of the triangles and their x, y and z coordinates respectively for a given structure.

The structure `geom` includes the following parameters.

geom.EdgesTotal	Integer	Total edges in the structure
geom.TrianglesTotal	Integer	Total triangles in the structure
geom.EdgeLength	[1,EdgesTotal]	Length of edge
geom.PointsFC	[3,TrianglesTotal]	Coordinates of center of metal faces
geom.TriP	[1,EdgesTotal]	Index on plus triangle
geom.TriM	[1,EdgesTotal]	Index on minus triangle
geom.VerP	[1,EdgesTotal]	Free vertex of plus triangle
geom.VerM	[1,EdgesTotal]	Free vertex of minus triangle
geom.AreaF	[1,TrianglesTotal]	Area of faces
geom.SizeF	[1,TrianglesTotal]	Effective triangle size
geom.IndexF	Integer	Order of integration for faces
geom.PointsF	[3,TrianglesTotal*IndexF]	Integration points of the Gaussian formulae
geom.PointsFRho	[3,IndexF,3,TrianglesTotal]	Vector drawn from the free vertex to the Gaussian points.
geom.PointsFCRho	[3,3,TrianglesTotal]	Vector drawn from the free vertex to the center of the face.
geom.W	[1,IndexF]	Weights of the points obtained during Gaussian subdivision
geom.MM00	[1,TrianglesTotal]	self-integral $1/r$ without V^2
geom.MM01	[1,TrianglesTotal]	self-integral r without V^2
geom.MM1	[3,3,TrianglesTotal]	self-integral $\frac{r_i r_j}{r}$ without V^2
geom.MM2	[3,3,TrianglesTotal]	self-integral $r_i r_j$ without V^2
geom.MM3	[3,3,TrianglesTotal]	self-integral $r_i r_j \cdot r$ without V^2
geom.P	[3,TrianglesTotal]	Coordinates of the vertices of triangles
geom.t	[4,TrianglesTotal]	Vertices of triangles

2.3 CALCULATE THE IMPEDANCE MATRIX

zmm

Computes the impedance matrix for pure metal structures.

Syntax:

$z = \text{zmm}(\text{geom}, \text{const}, \text{frequency})$

Description:

The function `zmm` returns matrix `ZMM` [`geom.EdgesTotal`, `geom.EdgesTotal`] which corresponds to the impedance matrix for pure metal. The inputs to the function are two structures `geom` and `const` and the `frequency` of operation in Hz. The structure `geom` defines all the parameters of the metal triangles including self integrals which is obtained from `rwgm` while the structure `const` defines the electromagnetic constants such as ϵ_0 , μ_0 , c (speed of light in vacuum) etc.

Theory:

$$Z_{mn} = + \frac{j\omega\mu_0}{4p} \iint_{S_m S_n} \vec{f}_m^S(\vec{r}) \cdot \vec{f}_n^S(\vec{r}') g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} - \frac{j}{4p\omega\epsilon_0} \iint_{S_m S_n} (\nabla_s \cdot \vec{f}_m^S(\vec{r})) (\nabla_s \cdot \vec{f}_n^S(\vec{r}')) g(\vec{r}, \vec{r}') d\vec{r}' d\vec{r} \quad (2)$$

where

$g = \exp(-jkR)/R$, $R = |\vec{r} - \vec{r}'|$ is the free-space Green's function (time dependency $\exp(j\omega t)$ is assumed everywhere).

μ_0 = absolute permeability of free space

ϵ_0 = absolute permittivity of free space

$f_n^S(\vec{r})$ is the basis function as explained in section 2.1

The detailed derivation of the impedance matrix is given in section A1 of Appendix A.

2.4 CALCULATE THE VOLTAGE DUE TO INCIDENT FIELD

voltage

Computes the voltage vector for pure metal structures.

Syntax:

[V] =voltage (const, geom, frequency)

Description:

The function **voltage** returns vector V [1, geom.EdgesTotal] which corresponds to the voltage vector for pure metal. The inputs to the function are two structures **geom** and **const** along with the frequency of operation. The structure **geom** defines all the parameters of the metal triangles including self integrals which is obtained from **rwgm** while the structure **const** defines the electromagnetic constants such as ϵ_0 , m_0 , c (speed of light in vacuum) etc.

Detailed explanation of the mathematical formulation is given in equations A9 and A10 of Appendix A.

2.5 CALCULATE THE CURRENT VECTOR

slv

Computes the current vector.

Syntax:

```
I = slv (z, v);
```

Description

The function `slv` returns the current vector `I` [1, geom.EdgesTotal]. The inputs to the function are the two matrices `z` [geom.EdgesTotal, geom.EdgesTotal] obtained from `zmm` and `v` [1, geom.EdgesTotal] obtained from `voltage`. The function uses LAPACK function `ZSYSV` (`zsysv_` for LINUX) for solving equations of the form $[z][I] = [v]$ where z is a complex symmetric matrix.

See also:

LAPACK and BLAS functions in MATLAB Help.

2.6 CALCULATE THE CURRENT DENSITY ON PATCH SURFACE

current

Computes and display the current density for every surface patch

Syntax:

[Current_, Current] =current (geom, I)

Description:

The function `current` returns vectors `Current_ [3, geom.TriangleTotal]` which corresponds to current density for patch centroids and `Current [3, total vertices of triangles in the structure]` which corresponds to the current density for vertex points. The inputs to the function are the structures `geom` and current `I`. The structure `geom` defines all the parameters of the metal triangles including self integrals which are obtained from `rwgm` while the current `I` is obtained using `slv` from the `Z` and `V` matrices.

The surface current density, $\vec{J}_s$ is expanded into basis function in the form

$$\vec{J}_s = \sum_{n=1}^N I_n \vec{f}_n^s(\vec{r}) \quad (3)$$

The theory is explained in detail along with the mathematical formulation in section A5 of the Appendix A.

2.7 CALCULATE THE RADIATED /SCATTERED FIELDS AND POYNTING VECTOR.

fields

Computes radiated/scattered electric and magnetic field and the Poynting vector for radiated/scattered field.

Syntax:

[Poynting, E, H]=field (const, geom, I, frequency, Points, Option)

Description:

The function *field* returns vectors

E [3, PointsS] Radiated/scattered electric field (complex vector at a point, V/m)

H [3, PointsS] Radiated/scattered magnetic field (complex vector at a point, A/m)

Poynting [3, PointsS] Poynting vector (W / m^2) for radiated/scattered field

where PointsS is the matrix of Points where the field is calculated

The inputs to the function are two structures **geom** and **const**. The structure **geom** defines all the parameters of the metal triangles including self integrals which is obtained from **rwgm** while the structure **const** defines the electromagnetic constants such as ϵ_0 , m_0 , c (speed of light in vacuum) etc. In addition to these, the other input parameters include the current **I** which is obtained using **slv**, **frequency** of operation in Hz, observation point where the field is to be measured and an additional parameter called **Option**. The value of option can be 0 or 1 depending on whether scattered field or total field is to be calculated.

Option = 0 Calculates scattered field at a point

Option = 1 Calculates total field at a point

The theory is explained in detail along with the mathematical formulation in section A5 of the Appendix A.

2.8 CALCULATE THE SURFACE FORCE DENSITY

force

Computes the force density for patch centroids and vertex points.

Syntax:

```
[Force_, Force] =force (geom., const, frequency, I, E,
                        H, option)
```

Description

The function field returns vectors

Force_ [3, geom.TriangleTotal] - surface force density for patch centroids

Force [3, total vertices of triangles in the structure] surface force density for vertex points.

The inputs to the function are two structures **geom** and **const**. The structure **geom** defines all the parameters of the metal triangles including self integrals which is obtained from **rwgm** while the structure **const** defines the electromagnetic constants such as ϵ_0 , m_0 , c (speed of light in vacuum) etc. In addition to these, the other input parameters include the current **I** which is obtained using **slv**, **frequency** of operation in Hz, the electric and magnetic fields obtained using **field** and an additional parameter called **option**. The value of **option** can be 0 or 1 depending on whether full force or Lorentz force is to be calculated.

option = 0 only Lorentz force $\vec{F} = \vec{J} \times \vec{B}$

option = 1 full force $\vec{F} = (\vec{J} \times \vec{B}) + E \cdot q$