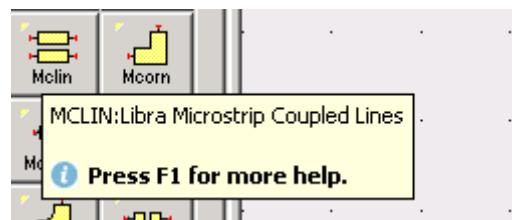


Lecture XIV ADS - I

In this lecture, we start to explain (at least try to) how the ADS is working. The explanation includes several distinct steps and exposure to several new concepts.

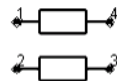
Part I ADS Libra

1. Many RF passives are precalculated in ADS, see Fig. 1, using a library called "Libra".

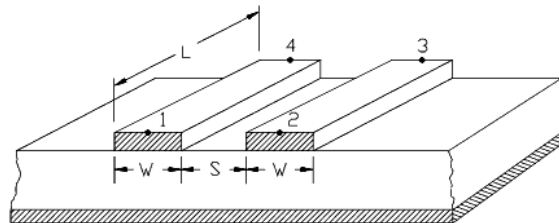


MCLIN (Microstrip Coupled Lines)

Symbol



Illustration



Available in ADS and RFDE

Fig. 1. A precalculated model of the coupled microstrips in ADS.

Now, what is Libra? Let us consider the example of two coupled microstrip lines shown in Fig. 1. If we refer to the paper

E. Hammerstad and O. Jensen, "Accurate models for microstrip computer-aided design," in IEEE MTT-S Inter. Microwave Symp. Dig., Washington, DC, 1980, pp. 407-409.

then we'll find there an analytical solution for the capacitance matrix (including even and odd mode capacitances and/or impedances) for two coupled microstrips. Such solutions are typically obtained using the method of conformal mapping (solving the Laplace equation for the electric potential analytically on the complex plane), as well as by interpolation of the available experimental and other analytical data.

As an example, the same solution has been programmed in the MATLAB script `imp_microstrip_coupled.m` to this lecture. No frequency-dependent dispersion has been included though. Also, the zero loss tangent of the substrate material was assumed.

It is now claimed that virtually any Libra element in ADS is a preprogrammed old (or not very old) analytical solution from one of the IEEE journals and/or a combination of those solutions. To prove this fact we could take a look at the corresponding help files. Also, the following HW problem is suggested:

HW#12 exercise #2

1. Using the MATLAB script `imp_microstrip_coupled.m` to this lecture find the odd/even mode impedances Z_e, Z_o for two coupled microstrips with the following parameters:

```
d          = 1.57e-3;      % Microstrip height above ground, m
w          = 0.5*d;        % Microstrip width, m
S          = 0.2*d;        % Trace separation, m
eps_r      = 10.0;         % Relative dielectric constant
```

The loss tangent of the substrate is zero.

2. Using the ADS Line Calculator (LineCalc) solve the same problem at

- A. 1 GHz
- B. 10 GHz

Assume the lossless dielectric substrate and the thickness of the metal lines of 1.4 mil. Furthermore, assume $\mu_r = 1$ and other default parameters. The line length may be arbitrary.

3. Find the error percentage between two solutions for every frequency. .

Part II ADS Momentum (MoM RF solver) - what exactly is behind?

1. Prerequisites of the MoM for RF problems

A. In contrast to FDTD and to FEM, the MoM operates in terms of currents/charges, but not in terms of the fields, which is more appealing to the broader ECE audience (in contrast to the Physics audience though). We've seen in Lecture #11 that electromagnetic radiation is caused by *both* charges and currents.

B. In a general case, these are the electric current density $\vec{J}$ of free charges with the units of A/m², and the volumetric electric charge density ρ of free charges with the units of C/m³.

C. Those volumetric quantities are usually reduced to the currents and charges on the metal surfaces (surface charges and currents) or to line currents on wires and, maybe, to conduction currents in a lossy dielectric. In a dielectric, the *volume polarization currents* are sometimes considered, and the *bound charges* on the dielectric interfaces.

D. The currents and charges on metal conductors are not given; they must be found from the MoM solution. The MoM does not solve the Maxwell's equations directly, it rather uses the boundary conditions for fields such as

$$\vec{E}_{\tan} = 0 \quad (*)$$

where *tan* denotes the tangential component of the *E*-field that is parallel to the metal surface, in order to obtain the closed-form equations for currents and/or charges. In order to do so, the field(s) must be expressed in terms of charges and currents first.

E. A good preliminary example has been the capacitance calculation for 2D transmission lines where we obtained an integral equation for the charges on the line using the boundary conditions for the electric potential (voltage) - see Lecture #6.

F. An ideologically similar procedure is applied to the 3D RF case, but it is a bit more involved. Thus, the first step would be to "solve" the Maxwell's equations with sources - charges and currents - in order to express the fields in terms of charges/currents. Below, we present the corresponding method in a general 3D form.

2. Maxwell's equations with sources (charges and currents)

Consider an arbitrary (inhomogeneous) medium with the electric permittivity ε having the units of F/m and with the magnetic permeability μ having the units of H/m. We assume that there is no (fictitious) magnetic current density.

The *driving sources* are given by the (volumetric) electric current density $\vec{J}$ of free charges with the units of A/m², and by the volumetric electric charge density ρ of free charges with the units of C/m³. The free charges are the free electrons in a metal or free electrons and/or holes in a semiconductor. Maxwell's equations for the electric field (or the electric field intensity) $\vec{E}$ [V/m] and for the magnetic field (or the magnetic field intensity) $\vec{H}$ [A/m] have the form

$$\text{Ampere's law modified by displacement currents} \quad \varepsilon \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{H} - \vec{J} \quad (1a)$$

$$\text{Faraday's law} \quad \mu \frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E} \quad (1b)$$

$$\text{Gauss' law for electric field} \quad \nabla \cdot \varepsilon \vec{E} = \rho \quad (1c)$$

$$\text{Gauss' law for magnetic field (no magnetic charges)} \quad \nabla \cdot \mu \vec{H} = 0 \quad (1d)$$

$$\text{Continuity equation for the electric current} \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \quad (1e)$$

Note: If $\vec{J}$ and ρ are equal to zero, a solution to Eqs. (1) is given by $\vec{E} = \vec{H} = 0$. In other words, there are no fields in free space without the *sources*: the electric charges and the electric currents. On the other hand, if $\vec{E} = \vec{H} = 0$ then $\vec{J}$ and ρ are equal to zero too.

Eq. (1d) has a general solution in the form

$$\nabla \cdot \mu \vec{H} = 0 \Rightarrow \mu \vec{H} = \nabla \times \vec{A} \quad (2)$$

where $\vec{A}$ is a new function - the magnetic vector potential with the units of V·s/m. Eq. (2) follows from the vector calculus identity

$$\nabla \cdot (\nabla \times \vec{A}) = 0, \quad (3)$$

which is valid for any function $\vec{A}(\vec{r}, t)$.

3. Solution to Maxwell's equations in terms of electric/magnetic potentials

We will now obtain a general solution to Eqs. (1) in a homogeneous ($\mu = \mu_0 = \text{const}$, $\varepsilon = \varepsilon_0 = \text{const}$) unbounded medium (air), due to an arbitrary current and charge distribution. This solution describes the behavior of any high-frequency circuit, no matter what is its particular shape or configuration.

We start with Eq. (1b) in the homogeneous medium

$$\mu_0 \frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E} \quad (4a)$$

and use Eq. (2) for the magnetic vector potential

$$\mu_0 \vec{H} = \nabla \times \vec{A} \quad (4b)$$

This gives

$$\nabla \times \left[\vec{E} + \frac{\partial \vec{A}}{\partial t} \right] = 0 \quad (4c)$$

A general solution to Eq. (4c) has the form

$$\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \quad (4d)$$

since $\nabla \times (\nabla \varphi) = 0$. Eq. (4d) is reduced to the well-known static result when the time dependence is absent. Eqs. (4b) and (4d) define the fields in terms of two potentials: the (so far undefined) *electric scalar potential* φ and the *magnetic vector potential* $\vec{A}$. Now, we need to find the potentials themselves, i.e. express them through charges/currents.

4. Equations for potentials (Lorentz gauge)

We will use the Ampere's law Eq. (1a) and the so-called *Lorentz gauge* to derive the equations for both potentials. We will eventually express both fields in Eq. (1a) in terms of the potentials.

Taking the curl of both sides of Eq. (4b) gives

$$\mu_0 \nabla \times \vec{H} = \nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \quad (5a)$$

We again have used the vector identity $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$. We plug Eq. (5a) into the Maxwell's equation (1a) and obtain

$$\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} - \mu_0 \vec{J} \quad (5b)$$

As a next step, we use Eq. (4d) $\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t}$ to express the electric field. Substitution into Eq. (5b) gives

$$\nabla^2 \vec{A} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \left[\nabla(\nabla \cdot \vec{A}) + \varepsilon_0 \mu_0 \nabla \frac{\partial \varphi}{\partial t} \right] \quad (5c)$$

The term in square brackets has the form

$$\nabla \left[\nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} \right] \quad (5d)$$

We use the Lorentz gauge, that is

$$\nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} = 0 \Rightarrow \frac{\partial \varphi}{\partial t} = -\frac{1}{\varepsilon_0 \mu_0} \nabla \cdot \vec{A} \quad (5e)$$

Question: What is the Lorentz gauge?

Answer: It follows from Eq. (2) that the substitution $\vec{A} \rightarrow \vec{A} + \nabla \psi$ will be the solution, for an arbitrary function ψ . This function may be chosen so that Eq. (5e) is satisfied.

Eqs. (5c) and (5e) are the required equations for the potentials:

$$\nabla^2 \vec{A} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} \quad (6a)$$

$$\frac{\partial \varphi}{\partial t} = -\frac{1}{\epsilon_0 \mu_0} \nabla \cdot \vec{A} \quad (6b)$$

Only the magnetic vector potential needs to be found from Eq. (6a) - the inhomogeneous wave equation. The electric scalar potential is then trivially obtained from Eq. (6b). After the potentials are found, the fields are given by Eqs. (4b) and (4c), i.e.

$$\mu_0 \vec{H} = \nabla \times \vec{A} \quad (6c)$$

$$\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \quad (6d)$$

Well, one step is still remaining: we need to solve Eq. (6a). To do so we will consider time-harmonic fields.

5. Phasor form of Eqs. (6)

Eqs. (6) or the original Maxwell's equations for a time-harmonic field in a uniform medium where $\mu = \mu_0 = \text{const}$, $\epsilon = \epsilon_0 = \text{const}$ are written in the phasor form. In the phasor form or in frequency domain, one introduces the time dependence in the form of a factor $\exp(j\omega t)$ for all the field quantities. The basic prerequisite for such a substitution is the linearity of Maxwell's equations or Eqs. (6). The factor $\exp(j\omega t)$ is then cancelled in every equation. If the phasors are denoted by the same letters as the original fields and sources in time domain, one has from Eqs. (6)

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu_0 \vec{J}, \quad k = \sqrt{\epsilon_0 \mu_0} \omega = \frac{\omega}{c_0} \quad (7a)$$

$$\varphi = -\frac{1}{j\omega \epsilon_0 \mu_0} \nabla \cdot \vec{A} \quad (7b)$$

$$\mu_0 \vec{H} = \nabla \times \vec{A} \quad (7c)$$

$$\vec{E} = -\nabla \varphi - j\omega \vec{A} \quad (7d)$$

Here, $c_0 = 1/\sqrt{\mu_0 \epsilon_0}$ is the speed of light and $k = \frac{\omega}{c_0}$ is the wavenumber.

The only point of concern is Eq.(7b): one gradient operation is necessary to find the electric potential and another - to find the electric field from the electric potential in Eq. (7d). This circumstance is not very convenient from the numerical point of view. Plus, we did not yet use the continuity equation, i.e. Eq. (1e). It may be written in the phasor form as follows

$$j\omega\rho + \nabla \cdot \vec{J} = 0 \quad (7e)$$

6. Mixed-potential formulation

Taking the gradient of both sides of Eq. (7a) one has

$$\nabla^2 (\nabla \cdot \vec{A}) - k^2 (\nabla \cdot \vec{A}) = -\mu_0 (\nabla \cdot \vec{J}) \quad (8a)$$

Now, $\nabla \cdot \vec{A}$ can be replaced by φ from Eq. (7b) and $\nabla \cdot \vec{J}$ can be replaced by charge density ρ from Eq. (7e). This yields a separate equation for the electric potential

$$\nabla^2 \varphi + k^2 \varphi = -\frac{\rho}{\epsilon_0} \quad (8b)$$

Eqs. (7) and Eq. (8b) form the so-called mixed-potential formulation of the Maxwell's equations. This formulation is very intuitive. It is also convenient from the numerical point of view. We will summarize the resulting equations in the mixed-potential form one more time for convenience

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu_0 \vec{J} \quad (9a)$$

$$\nabla^2 \varphi + k^2 \varphi = -\frac{\rho}{\epsilon_0} \quad (9b)$$

$$j\omega\rho + \nabla \cdot \vec{J} = 0 \quad (9c)$$

$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A} \quad (9d)$$

$$\vec{E} = -\nabla \varphi - j\omega \vec{A} \quad (9e)$$

7. Solution to Eqs. (9)

The inhomogeneous wave equation (9a) and a very similar equation (9b) have been studied by mathematicians for many years. About 100 years ago or so it has been found that a general solution to those equations in an unbounded medium, with the automatic field decay at infinity, has the form of an integral

$$\vec{A}(\vec{r}) = \mu_0 \iiint \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi|\vec{r} - \vec{r}'|} \vec{J}(\vec{r}') d\vec{r}' \quad (10a)$$

$$\phi(\vec{r}) = \iiint \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi\epsilon_0|\vec{r} - \vec{r}'|} \rho(\vec{r}') d\vec{r}' \quad (10b)$$

The structure of those integrals is of note. We integrate over $\vec{r}'$ (the *integration variable*), but the result depends on $\vec{r}$ (the *observation variable*). The integrand includes two parts: the source current $\vec{J}(\vec{r}')$ or the source charge $\rho(\vec{r}')$ and a function $\exp(-jk|\vec{r} - \vec{r}'|)/|\vec{r} - \vec{r}'|$, which is called the *Green's function* or the *fundamental solution* to Maxwell's equations.

Eqs. (10) complete the solution of Maxwell's equations. If the current and charge distributions for an antenna are given, we find integrals (10) first. Then we calculate the radiated fields from Eqs. (9). Eqs. (9) and (10) are valid at any frequency, all the way from DC to $f \rightarrow \infty$. Again, the function

$$G(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi\epsilon_0|\vec{r} - \vec{r}'|} \quad (11)$$

is the *Green's function* or the fundamental solution to the Helmholtz equation *in an unbounded space*. Eqs. (1) are valid at any frequency, all the way from DC to $f \rightarrow \infty$, and for any circuit geometry.

Note: Similar Green's functions exist for an infinite metal ground plane, infinite metal ground plane with a dielectric substrate, etc. Discuss- give some examples.

Part III ADS Momentum -integral equation and basis functions

1. Boundary conditions

The solution given by Eqs. (9e), (10a,b) should be used along with the boundary conditions on the metal surface.

Note: The boundary conditions at infinity (free radiation at infinity) are automatically taken into account by the free-space Green's function $G(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi\epsilon_0|\vec{r} - \vec{r}'|}$. This is a major advantage of the integral equation method, that we will pursue, as compared to other antenna modeling tools - FDTD and FEM.

On a metal surface S of a *perfect* electric conductor, the tangential component of the total electric field vanishes,

$$\vec{E}_{\text{tan}} = 0 \quad (12)$$

where *tan* denotes the tangential component of the E -field that is parallel to the surface. Otherwise, an infinite current would flow there, due to the infinite medium conductivity. The normal component of the electric field could have any nonzero values associated with the surface charges.

2. Integral equation method

Why do we need integral equations? Well, Eqs. (9d,e) express the fields in terms of current and charge distributions $\vec{J}(\vec{r})$, $\rho(\vec{r})$. But do we know the currents and charges on the metal surface? The answer is no! In practice, the current on the metal surface is variable.

We have solved Maxwell's equations. We cannot obtain more information from them. However, we did not check the boundary conditions yet. We substitute Eq. (9e) into Eq. (12) and obtain

$$\left(j\omega\vec{A} + \nabla\phi\right)_{\text{tan}} = 0 \quad \text{everywhere on the metal surface except for the ports} \quad (13a)$$

where

$$\vec{A}(\vec{r}) = \mu_0 \int_{\mathbb{R}^3} G(\vec{r}, \vec{r}') \vec{J}(\vec{r}') d\vec{r}', \quad \phi(\vec{r}) = \frac{1}{\epsilon_0} \int_{\mathbb{R}^3} G(\vec{r}, \vec{r}') \rho(\vec{r}') d\vec{r}', \quad j\omega\rho + \nabla \cdot \vec{J} = 0 \quad (13b)$$

Eqs. (13) are the celebrated integral equations of electromagnetics solved by the Method of Moments (MoM). They allow us to find the unknown current and charge distributions on the metal surface. After that operation, the fields are found using Eqs. (9d,e).

The method of integral equation is the only method that explicitly deals with voltages, currents, and charges. The FEM method and the FDTD method deal with fields.

3. Basis functions

Eqs. (13) have to be solved using a *mesh* of small segments, triangles, rectangles, tetrahedra, or etc. Basis functions need to be defined on the mesh. These basis functions must support *both current and charges*; they cannot be therefore the piecewise-constant basis functions for the current.

The best known and most popular vector basis functions are the *rooftop* basis function (also called Rao-Wilton-Glisson or RWG basis functions) introduced in 1982 for metal patches and in 1960s-1970s for metal wires. The idea is shown in Fig. 2 (for a metal wire and the line current on the wire).

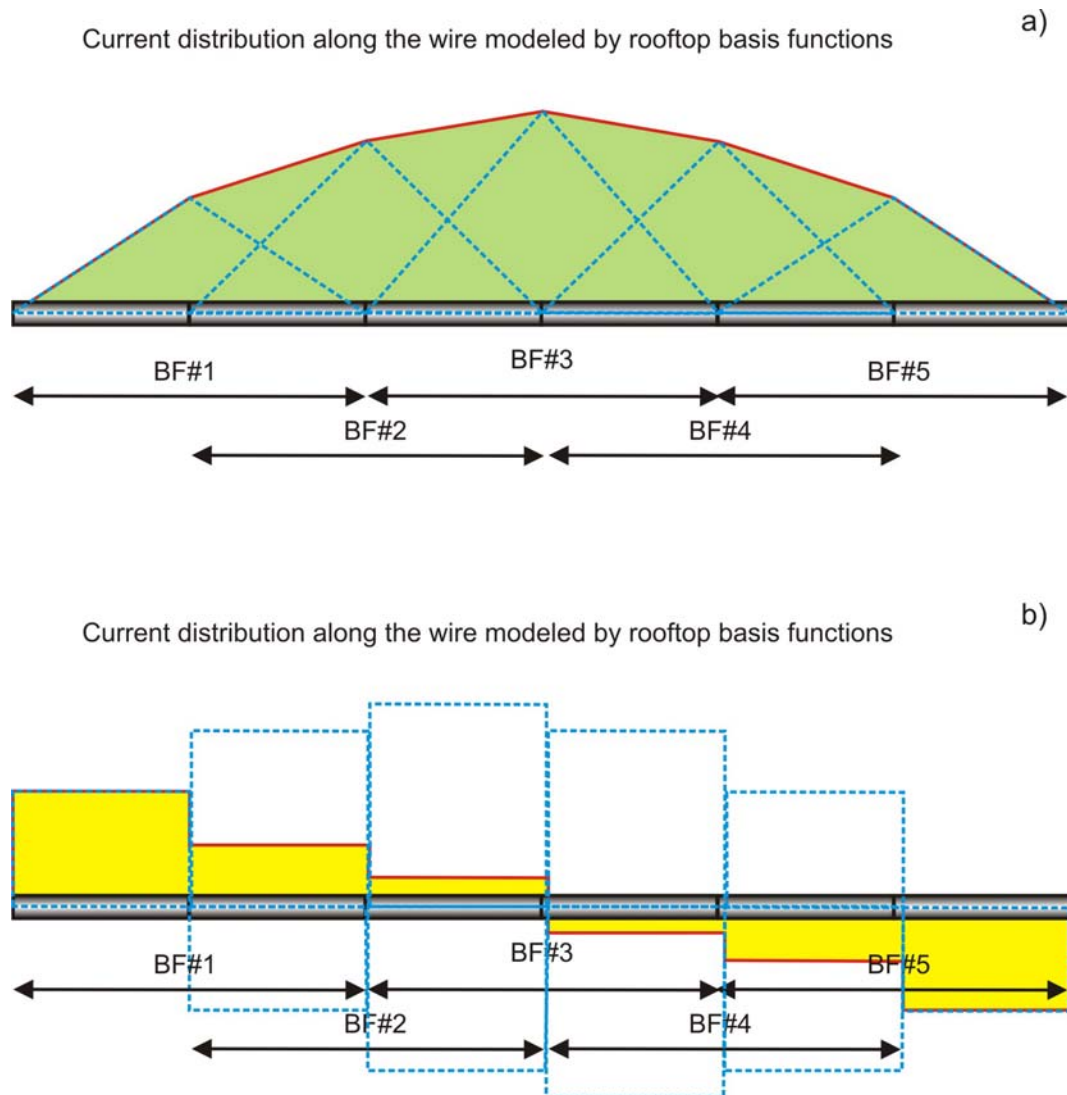


Fig. 2. Rooftop basis functions - the concept. Top - current expansion; bottom - charge expansion.

Note: The following important remarks could to be made:

1. The rooftop basis functions are primarily formulated for the electric current.
2. Every basis function is defined on two adjacent segments in Fig. 2.
3. Every basis function is piecewise linear for currents; it resembles a rooftop.
4. Any current distribution along a wire is reasonably well described by the rooftop basis functions. The accuracy improves when the mesh is refined.
5. Every basis function is piecewise constant for charges - see Fig. 2b. This is because the charge is essentially the derivative of the current - see Eq. (9c) and then apply it in the 1D case.
6. Any charge distribution along a wire is reasonably well described by the rooftop basis functions using the staircase approximation. The accuracy improves when the mesh is refined.
7. The wire may be bent (!); the same expansion into rooftop basis functions takes place.
8. The segments do not have to be of the same size!

Note: NEC (Numerical Electromagnetic Code) is the MoM software; it is using the rooftop basis function on wires and the KCL for wire junctions. Every surface is represented as a dense metal mesh in NEC.

In a similar way, the rooftop basis functions could be defined for surface patches - triangles and rectangles. Fig. 3 shows the concept. The ADS is using exactly the rooftop basis functions on rectangles and triangles, also generalized for polygonal domains (T-shaped, etc.):

Knockaert et al., "Generalized polygonal Basis Functions for Electromagnetic Simulation of Complex Geometrical Planar Structures," in 2001 IEEE MTT-S Digest, pp. 1249-1252.

Other application notes may be used too.

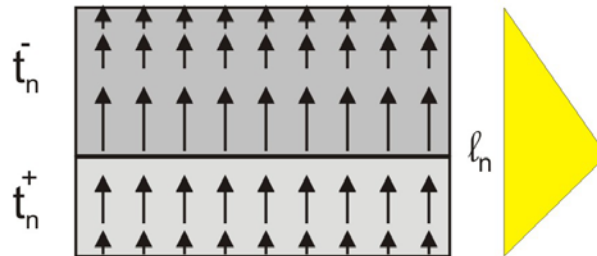
A similar rooftop concept also applies to tetrahedra. As an example, Fig. 4 shows the figure adopted from Ref.

A. W. Glisson et al, "Physically-Based Approximation of Electromagnetic field Quantities," IEEE-APS 2002, pp. 78-81.

It looks like that the rooftop basis functions are rather general. Are they? Discuss the problems (2D geometry representation) during the lecture.

c)

Current distribution for the basis function on two rectangles



b)

Current distribution for the basis function on two triangles

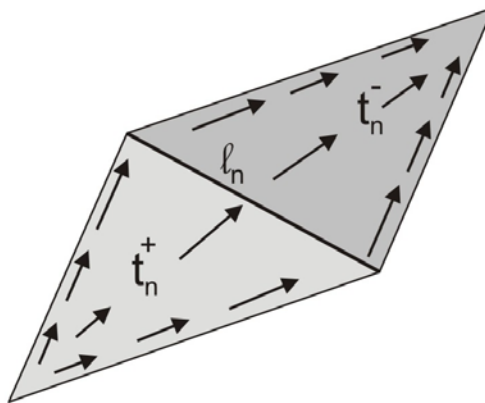


Fig. 3. Rooftop basis functions - the generalization to surface patches.

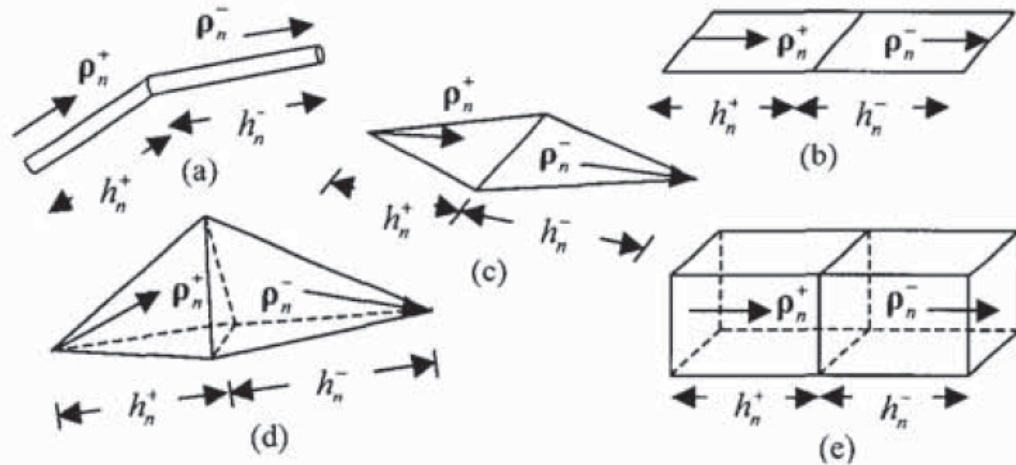


Figure 1. Generalized rooftop bases for (a) wire and cylinder, (b) rectangular, (c) triangular, (d) tetrahedral, and (e) brick elements.

Fig. 4. Generalization of the rooftop basis functions - Fig. 1 from Ref.: [A. W. Glisson et al, "Physically-Based Approximation of Electromagnetic field Quantities," IEEE-APS 2002, pp. 78-81.]