

Physically-Based Approximation of Electromagnetic Field Quantities

A.W. Glisson

Dept. of Electrical Engr.
University of Mississippi
University, MS 38677
aglisson@olemiss.edu

S. M. Rao

Dept. of Electrical Engr.
& Comp. Engr.
Auburn University
Auburn, AL 36849
rao@eng.auburn.edu

D. R. Wilton *

Dept. of Electrical
& Comp. Engr.
University of Houston
Houston, TX 77204.
wilton@uh.edu

1. Introduction

Mathematical and numerical analyses are necessary for analyzing error levels and convergence behavior of numerical algorithms, but they can be of limited usefulness in selecting or developing good representations for field or equivalent current quantities. The choice or development of basis/expansion functions is often best guided by applying physical principles. This presentation summarizes the development of various field and current approximations, and discusses principles guiding their development. The focus is on bases that model equivalent currents for integral equations solved by the method of moments.

2. Currents on Wires and Cylinders

With simplicity as a goal, early wire models used piecewise constant (pulse) functions to model both current and charge [1]. However, since the current distribution on a wire becomes sinusoidal in the limit of zero radius, more sophisticated schemes used piecewise sinusoidal (PWS) basis functions to expand the current modeling and to test the integral equation. As one might expect, PWS bases work extremely well for wires of resonant length. Between resonances, however, the PWS scheme converges no faster than the piecewise linear (PWL or "triangle") expansion scheme, to which it reduces in the limit of electrically small subdomains [2]. The PWL expansion scheme also works well for modeling circumferentially directed surface currents on cylindrical geometries. Among its desirable features are the following:

- When wire or cylinder cross sections are modeled using PWL segments, it is quite natural to approximate the current as PWL with interpolation points at the segment endpoints.
- PWL bases assure continuity of the current, but permit slope discontinuities at sources, bends, and radius discontinuities. Current continuity assures that no non-physical line charges appear at segment boundaries.
- PWL representations can exactly enforce vanishing current conditions at wire or surface boundaries. In a piecewise constant scheme, the effective vanishing point of the current, as determined by extrapolation, is often slightly shorter or longer than the actual geometry, thus reducing the approach's convergence rate [1].

Current components parallel to and invariant along a cylinder's axis do not have an accompanying surface charge density, but the current is singular near any surface edges. Extensive numerical experimentation has shown that either 1) the current model should incorporate the edge singularity or 2) the testing scheme should not emphasize fields near edges, since they are inherently incorrect [3].

3. Surface and Volumetric Currents

Early schemes for modeling conducting plates used rectangular subdomains with two orthogonal, constant current components in each subdomain. Though it works reasonably well with the magnetic field integral equation (MFIE), on open metallic plates the approach does not correctly capture the plate's effective area and does not model the current well at structure discontinuities. Furthermore, to avoid handling the line charges that arise in the electric field integral equation (EFIE), generalizations of the Hallén approach for wires may be required. The use of "rooftop" basis functions to model currents on plate structures obviates many of these difficulties [4]. Rooftop bases generalize and share the same properties as PWL bases in wire and cylinder modeling. For rectangular subdomains on planar surfaces, they also have the following features:

- The staggering of charge and current interpolation points allows the continuity equation to be satisfied in either differential or integral form; indeed, the surface field interpolation points coincide with their corresponding positions in a Yee lattice when the latter is collapsed onto a planar surface.
- They can be derived starting from a piecewise constant charge representation by integrating the continuity equation and requiring that the bases interpolate the current at subdomain boundaries.
- Rooftop bases permit an exact Helmholtz decomposition of surface currents into curl- and divergence-free parts (loops and trees/stars). This is a consequence of their underlying piecewise constant charge representation.

Bodies of revolution (BOR) illustrate particularly well the importance of using a consistent charge representation [4]. If triangle functions are used to represent both components of current along the generating arc of a BOR, different charge representations result from each component, causing spurious resonances to occur in EFIE formulations. The use of linear longitudinal and constant azimuthal current representations produces a consistent piecewise constant charge representation and eliminates the fictitious resonances.

For modeling curved surfaces and boundaries, triangular subdomains are often used, but the node-based approach pioneered by the finite element community cannot be used to define surface current bases for use with the EFIE:

- The nodes occur at surface model corners—both actual and model-induced—where tangential components cannot be defined.
- Current continuity across element boundaries cannot easily be enforced, resulting in the appearance of nonphysical line charges and an excessive number of degrees of freedom per element.

However, a piecewise constant charge representation may be used as a starting point to obtain bases that interpolate the normal current component at a triangle's edges. Taking a cue from rooftop bases, one assumes a constant charge density and a current basis having a component normal to one triangle edge and parallel to the other two. This suggests a basis of the form $\mathbf{f} = f_\rho \hat{\rho}$ which is directed radially from or toward the vertex opposite the edge with a normal component. Placing a local polar coordinate origin at the opposite vertex and using the continuity equation, we have

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho f_\rho) = C_1 \Rightarrow f_\rho = \frac{1}{2} \rho C_1 + \frac{C_2}{\rho}, \quad (1)$$

or $\mathbf{f} = \hat{\mathbf{p}}(\rho/h) = \rho/h$ where h is the triangle's height relative to the edge and serves to normalize the normal component to unity. A similar definition, but with oppositely directed vector, applies in the adjacent triangle sharing the edge. This representation appears implicitly in [5] and explicitly in [6]. It not only incorporates the PWL wire, cylinder, and rooftop bases described, but also extends to incorporate volumetric brick, tetrahedral [7], and wedge elements. This *generalized rooftop basis function* may be expressed as

$$\Lambda_n(\mathbf{r}) = \begin{cases} \frac{\rho_n^+}{h_n^+}, & \mathbf{r} \in D_n^+ \\ \frac{\rho_n^-}{h_n^-}, & \mathbf{r} \in D_n^- \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where ρ_n^+ is the vector to $\mathbf{r}$ from the vertex, edge, or face opposite a boundary of subdomain D_n^+ through which a current of unit density passes, and h_n^+ is the height of the subdomain relative to that boundary. These quantities are similarly defined in the adjacent subdomain D_n^- sharing the boundary, but with oppositely directed vectors (see Fig. 1).

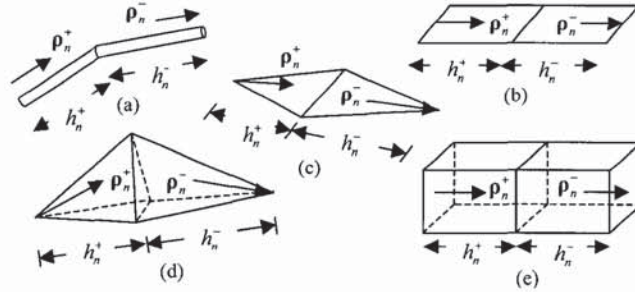


Figure 1. Generalized rooftop bases for (a) wire and cylinder, (b) rectangular, (c) triangular, (d) tetrahedral, and (e) brick elements.

4. Wire-to-Surface Junctions

The procedure described can also be used to model current in a triangle with a wire attached at its vertex, and results in the junction basis

$$\Lambda_n^j(\mathbf{r}) = K \left(1 - \frac{h^2}{(\mathbf{p} \cdot \hat{\mathbf{u}})^2} \right) \Lambda_n(\mathbf{r}),$$

where K appropriately normalizes the total current entering the wire, and $\hat{\mathbf{u}}$ lies in the plane of the triangle and is normal to the edge opposite the attachment vertex. The first term in the junction basis—an ordinary triangular surface patch basis function—produces a constant charge density. The second term is divergenceless and cancels the normal component resulting from the first term at the edge opposite the junction vertex. It also provides the $1/\rho$ current variation

expected at a wire-to-surface junction [8]. In using these bases, special considerations must be observed to ensure a well-conditioned problem.

5. Loop Star/Tree Basis Decompositions

Any divergenceless current distribution is a homogeneous solution of the EFIE at zero frequency. Piecewise constant charge representations ensure that discrete, divergenceless loops of current exist. While the EFIE remains unstable at sufficiently low frequencies, removing current loops from the original basis set allows the divergenceless current components to be treated separately. It also gives a partial Helmholtz decomposition of the current and allows the EFIE to be separated into loosely coupled, specially treated quasi-electrostatic and quasi-magnetostatic parts [9].

6. Conclusions

Robust bases for use with the EFIE can be developed by considering the underlying physics of the surface fields being modeled. The low order basis functions described not only satisfy these physical properties, but also form building blocks for higher order [10] and singular bases [11] that even more closely model the physics.

References

- [1] R. F. Harrington, "Matrix methods for field problems," *Proc. IEEE*, vol. 55, pp. 136–149, Feb. 1967.
- [2] C. M. Butler and D. R. Wilton, "Analysis of various numerical techniques applied to thin-wire scatterers," *IEEE Trans. Antennas Propagat.*, vol. 23, pp. 534–540, July 1975.
- [3] D. R. Wilton and S. Govind, "Incorporation of edge conditions in moment method solutions," *IEEE Trans. Antennas Propagat.*, vol. 25, pp. 845–850, November 1977.
- [4] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 593–603, Sept. 1980.
- [5] P. A. Raviart and J. M. Thomas, "A mixed finite element method for second order elliptic problems," in *Mathematical Aspects of Finite Element Methods*, A. Dold and B. Eckmann, Eds., New York: Springer, 1977.
- [6] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 30, pp. 409–418, May 1982.
- [7] D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 77–85, Jan. 1984.
- [8] S. U. Hwu, D. R. Wilton, and S. M. Rao, "Electromagnetic scattering and radiation by arbitrary conducting wire/surface configurations," *1988 IEEE Int. Antennas Propagat. Symp. Dig.* vol. 26, pp. 890–893, June 1988.
- [9] J. S. Lim, S. M. Rao, and D. R. Wilton, "A novel technique to calculate the electromagnetic scattering by surfaces of arbitrary shape," *Proc. URSI Radio Sci. Meet. Dig.*, Ann Arbor, MI, p. 322, June 1993.
- [10] R. D. Graglia, D. R. Wilton, and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. AP-45, pp. 329–342, Mar. 1997.
- [11] W. J. Brown and D. R. Wilton, "Singular basis functions and curvilinear triangles in the solution of the electric field integral equation," *IEEE Trans. Antennas Propagat.*, vol. AP-47, pp. 347–353, Feb. 1999.