

Multilevel Fast Multipole Method Solution of Volume Integral Equations Using Parametric Geometry Modeling

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Abstract—We present a multilevel fast multipole method (MLFMM) solution for volume integral equations dealing with scattering from arbitrarily shaped inhomogeneous dielectrics. The solution accuracy, convergence, computer time and memory savings of the method are demonstrated. Previous works have employed the MLFMM for impenetrable targets. In this paper, we integrate the MLFMM with the volume integral equation method for scattering by inhomogeneous targets. Of particular importance is the use of curvilinear elements for better volume representation and the use of simple basis functions for ease of parallelization.

Index Terms—Conformal geometry modeling, electromagnetic scattering by nonhomogeneous media, fast algorithms, fast multipole methods, integral equations, method of moments (MoM).

I. INTRODUCTION

SCATTERING by dielectric and composite volumes is of significant interest for radar cross section (RCS) and electromagnetic compatibility/electromagnetic interference (EMC/EMI) evaluation of modern composite vehicles and structures. Over the past three decades several authors considered the scattering by dielectric volumes using the method of moments (MoM) [1]. Among them, Livesay and Chen [2] employed brick elements and piecewise-constant basis functions for modeling the dielectric volume and Schaubert *et al.* [3] employed tetrahedral elements and linear basis functions (similar to those used in finite element formulations [4]). However, because the number of unknowns needed to formulate the moment method for volumetric dielectrics quickly grows to many thousands, even for a structure as small as $1\lambda^3$, the method is difficult to use for practical applications. An alternative is to use a boundary integral formulation for dielectrics [5], but this is only applicable to piecewise-homogeneous structures. To alleviate the $O(N^3)$ central processing unit (CPU) bottleneck for modeling dielectric volumes, k -space methods utilizing iterative solution approaches such as the conjugate gradient FFT method (CGFFT) were introduced in the late 1980s [6]–[9]. These latter methods employ the FFT to reduce storage and CPU requirements down to $O(N)$ and

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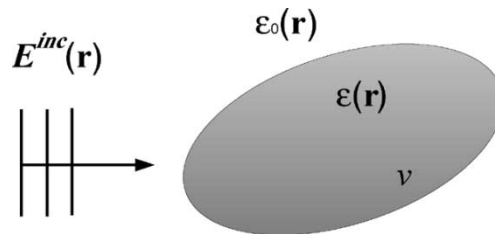


Fig. 1. Geometrical layout for VIE formulation.

$O(N \log N)$, respectively. However, by virtue of the FFT, they lack accuracy in modeling curvilinear dielectric structures. This is particularly so for high contrast dielectrics where, as demonstrated in [10], extremely high sampling is required for accurate solutions without use of conformal elements.

In the 1990s, the finite element-boundary integral (FE-BI) method [4], [11], [12] became a popular alternative for modeling inhomogeneous dielectric regions. Nevertheless, direct integral methods lead to less numerical error accumulation when dealing with large material volumes. With the recent success of fast methods such as the multilevel fast multipole method (MLFMM) [14], [15], it is important to re-examine volume integral methods as an alternative to modeling inhomogeneous volumetric structures (see [16] for an application of volume integral equations (VIEs) to indoor radio wave propagation).

In this paper we employ curvilinear hexahedral elements for conformal modeling of dielectric structures and introduce a MLFMM solution for VIEs. It is demonstrated that even piecewise-constant basis functions provide excellent accuracy at a nominal sampling rate. Results are given to show the accuracy of the method as well as CPU and memory savings. We find that piecewise-constant bases are particularly attractive for domain decomposition and ease of parallelization.

II. VIE FORMULATION

Consider the geometry shown in Fig. 1. It refers to a dielectric volume, possibly inhomogeneous, and we are interested in evaluating the scattered field from such a structure due to an excitation $\mathbf{E}^{\text{inc}}(\mathbf{r})$. The pertinent integral equation is [3]

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \int_v dv' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot (k^2(\mathbf{r}') - k_0^2) \mathbf{E}(\mathbf{r}') \quad (1)$$

where v is over the dielectric domain, $k(\mathbf{r}) = k_0 \sqrt{\epsilon_r(\mathbf{r})}$ is the wavenumber inside the inhomogeneous medium, and $\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$

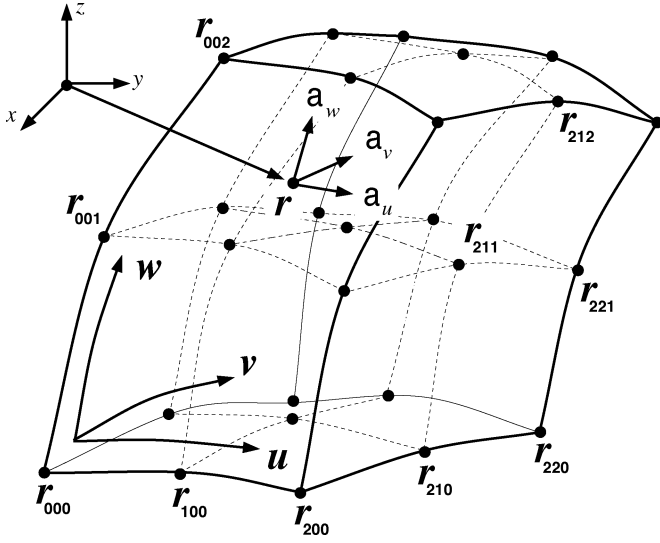


Fig. 2. Curved hexahedral element.

is the free-space dyadic Green's function. To solve for $\mathbf{E}$ using the MoM [17], we introduce the expansion

$$\mathbf{E}(\mathbf{r}) \approx \sum_{j=1}^N a_j \mathbf{e}_j(\mathbf{r}) \quad (2)$$

and apply Galerkin's method to get

$$\langle \mathbf{e}_i(\mathbf{r}), \mathbf{E}(\mathbf{r}) \rangle = \langle \mathbf{e}_i(\mathbf{r}), \mathbf{E}^{\text{inc}}(\mathbf{r}) \rangle + \left\langle \mathbf{e}_i(\mathbf{r}), \int_v dv' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot (k^2(\mathbf{r}') - k_0^2) \mathbf{E}(\mathbf{r}') \right\rangle \quad (3)$$

for $i = 1, \dots, N$. This can be written as $[\mathbf{Z}]\{\mathbf{a}\} = \{\mathbf{b}\}$, with

$$\mathbf{Z}_{ij} = \langle \mathbf{e}_i(\mathbf{r}), \mathbf{e}_j(\mathbf{r}) \rangle - \left\langle \mathbf{e}_i(\mathbf{r}), \int_v dv' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot (k^2(\mathbf{r}') - k_0^2) \mathbf{e}_j(\mathbf{r}') \right\rangle \quad (4)$$

being the impedance matrix elements and

$$\mathbf{b}_i = \langle \mathbf{e}_i(\mathbf{r}), \mathbf{E}^{\text{inc}}(\mathbf{r}) \rangle \quad (5)$$

are the excitation elements.

Two key issues in the above formulation refer to the choice of the basis functions and the solution of the matrix system. Both of them are critical to the solution accuracy. Previous formulations have employed cubic [2] and tetrahedral [3] elements, both of which can cause inaccuracies in approximating the geometry. As discussed in [10], proper geometry approximation is important to maintain accuracy, particularly for high dielectric constants.

In this paper we propose the use of curvilinear elements as displayed in Fig. 2. These are second order shape functions but the field representation within each element can be chosen independent of the order of the element shape. That is, we can choose a piecewise-constant (in parametric space) or a piecewise-linear field representation within each cell. Here, we employ the piecewise-constant conformal basis functions which were found to

render easily parallelizable algorithms at the likely expense of higher density grids.

To define the basis functions, let us first begin by introducing the geometry of the volume element. The proposed 27-point hexahedral element can be defined in the parametric (u, v, w) space using the transformation

$$\mathbf{r}(u, v, w) = \sum_{n=0}^2 \sum_{p=0}^2 \sum_{q=0}^2 \mathbf{r}_{npq} L_{npq}(u, v, w) \quad (6)$$

where $\mathbf{r}_{npq}$ are the defining 27 points of the hexahedron and $L_{npq}(u, v, w)$ are the quadratic Lagrange interpolation functions in three parameters (u, v, w) . These coefficients can be constructed using the 27 constraints

$$\mathbf{r}\left(u = \frac{n}{2}, v = \frac{p}{2}, w = \frac{q}{2}\right) = \mathbf{r}_{npq} \quad (7)$$

where $n, p, q \in \{0, 1, 2\}$.

With the above geometry definitions, we now proceed to define the basis functions. For piecewise-constant bases, only three functions are needed and of importance is to define the vector direction of each function. Specifically, we choose the three vectors to be the covariant directions of the curvilinear hexahedron. They are (see Fig. 2)

$$\mathbf{a}_u(\mathbf{r}) = \frac{\partial \mathbf{r}}{\partial u}, \quad \mathbf{a}_v(\mathbf{r}) = \frac{\partial \mathbf{r}}{\partial v}, \quad \mathbf{a}_w(\mathbf{r}) = \frac{\partial \mathbf{r}}{\partial w}. \quad (8)$$

Ideally, we need three orthogonal components of the discretized unknown, i. e. the electric field intensity within the dielectric region. For the curvilinear hexahedral discretization used in this paper, these covariant basis vectors result in an almost orthogonal set of conformal directions provided the hexahedron is not severely distorted.

Using these covariant set of vectors, the piecewise-constant basis functions can be expressed as

$$\begin{aligned} \mathbf{e}^{(u)}(\mathbf{r}) &= \frac{1}{\sqrt{G}} \mathbf{a}_u(\mathbf{r}) \\ \mathbf{e}^{(v)}(\mathbf{r}) &= \frac{1}{\sqrt{G}} \mathbf{a}_v(\mathbf{r}) \\ \mathbf{e}^{(w)}(\mathbf{r}) &= \frac{1}{\sqrt{G}} \mathbf{a}_w(\mathbf{r}) \end{aligned} \quad (9)$$

where, the determinant of the metric tensor of the parametric transformation (6) is given by

$$G = \begin{vmatrix} g_{uu} & g_{uv} & g_{uw} \\ g_{vu} & g_{vv} & g_{vw} \\ g_{wu} & g_{wv} & g_{ww} \end{vmatrix} \quad g_{\eta\xi} = \frac{\partial \mathbf{r}}{\partial \eta} \cdot \frac{\partial \mathbf{r}}{\partial \xi} \quad (10)$$

in which, η and ξ represent any of the parameters u, v , and w . When expressed as above, the proposed basis functions reduce to those in [2] for rectangular finite elements. Thus, they can be considered as a generalization of those in [2]. We also remark that in finite element analysis, the contra-variant vectors [13] are typically employed so that the bases have small divergence (and constant curl). However, no requirement on the divergence or curl of the basis function is necessary for the VIEs. Thus, a covariant representation with piecewise-constant basis function is chosen for simplicity.

TABLE I
SURFACE MESH FOR A SPHERE

Δ	Quads	Triangles
0.250	150	300
0.200	216	432
0.150	384	768
0.100	864	1728
0.075	1536	3072
0.050	3750	7500
0.040	5766	11532

TABLE II
VOLUMETRIC MESH FOR A SPHERE

Δ	Hexas	Tetras
0.250	125	625
0.200	216	1080
0.150	512	2560
0.100	1728	8640
0.075	4096	20480
0.050	15625	78125
0.040	29791	148955

III. MLFMM

So far, the MLFMM has been applied in the context of surface integral equation (SIE) formulations both for impenetrable (PEC) target geometries [18] and FE-BI formulations of inhomogeneous penetrable targets [19]. In this paper, we apply the MLFMM to VIE formulations as outlined above. To point out the potential of MLFMM as applied to VIEs, let us consider a sphere of unit radius. In proceeding with the numerical SIE solution, a meshing package, such as MSC-PATRAN finite element mesher is first used to produce a surface mesh consisting of, say, quadrilateral elements. The size of the surface elements are determined by the wavelength of interest (the rule of thumb is 10 elements per linear wavelength) and the resulting numerical system dimension is proportional to the number of elements in the mesh. As a function of element size Δ , the data in Table I were produced from the MSC-PATRAN's mesher. When the same gridding package is used to generate a volumetric mesh using hexahedral elements, the number of resulting finite elements are given in Table II as a function of the specified maximum edge length. Two observations can be made between Tables I and II. First, the number of tetrahedra needed for the same discretization rate is much larger (five times larger) and thus tetrahedral meshing lead to many more unknowns. Second, in comparing Tables I and II, the number of unknowns grow dramatically for volume formulations. Specifically, for volume formulations the number of unknowns is proportional to $1/\Delta^3$ as compared to $1/\Delta^2$ for surface formulations. For high contrast dielectrics, $\Delta \approx \lambda_0/10\sqrt{\epsilon_r}$ must necessarily be smaller and this further

exacerbates the situation. Therefore, for VIE formulations, the problem size quickly grows to many thousands, even for a physically small geometry. To overcome the high computational demands of VIEs, one must resort to more efficient strategies, like the MLFMM for solving the MoM matrix.

The first step in implementing the MLFMM is to develop a multilevel grouping procedure for volume discretizations as done for SIE formulations. For our procedure, the scatterer is enclosed in a cube, which is then divided into eight subcubes, each of which is further divided into eight smaller subcubes, and so on, to form an oct-tree structure. For the finest level of subdivision, each basis function is associated with a cube and since we have three piecewise-constant basis functions in each hexahedron, we proceed to form the clustering in terms of elements rather than basis functions. All three basis functions are then associated with that cluster containing the element that supports them. This clustering approach is important for a parallel implementation since it allows for a physical domain decomposition of the computational space. Using this association, the signature functions for the basis functions are subsequently computed in reference to the center of the cube for each plane wave direction and stored in the computer memory. This computation differs from the surface formulation only in that the signature functions involve volume rather than surface integrals.

Once the signature functions for each basis function are computed and stored, the rest of the MLFMM algorithm is identical to that of a surface formulation [18]. However, the MLFMM parameters must be readjusted for optimal performance since the unknowns are distributed in a volume, rather than on a surface. Consequently, the cost of the MLFMM part in the matrix-vector product for the near-field section of the matrix will differ from that of the SIE MLFMM solver. A fine tuning of the implementation is also necessary if different clustering approaches must be employed, since the MLFMM performance relies heavily on the clustering setup. For example, the MLFMM parameters (i.e., the number of multipoles and the bandwidth of the near-field matrix) are predetermined using the size and neighborhood information of the clusters. Furthermore, clustering information must be utilized for the parallel implementation of the MLFMM solver to balance the computational load across the nodes. For example, for the oct-tree subdivision algorithm outlined above (for surface formulations), assuming a locally flat geometry, leads to $3 \times 3 = 9$ near clusters, whereas the number of far clusters is $6 \times 6 - 3 \times 3 = 27$ (see Fig. 3). For volumetric structures, the number of near clusters is $3 \times 3 \times 3 = 27$ and the number of far clusters is $6 \times 6 \times 6 - 3 \times 3 \times 3 = 189$. Due to a volumetric distribution of unknowns, a larger number of neighboring clusters is associated with a given cluster. Hence, the computational burden of VIEs may be as much as three times higher for the near-field matrix-vector product and eight times higher for the far-field portion compared to SIE-MLFMM implementations.

The core equation of the MLFMM algorithm is the spectral (multipole) representation of free-space Green's function. Consider the two level vector construct shown in Fig. 4. Decomposing $\mathbf{r}_{ij}$ as

$$\mathbf{r}_{ij} = \mathbf{r}_{im}^{(l+1)} + \mathbf{r}_{mm}^{(l)} + \mathbf{r}_{mm'}^{(l)} - \mathbf{r}_{m'm'}^{(l)} - \mathbf{r}_{jm'}^{(l+1)} \quad (11)$$

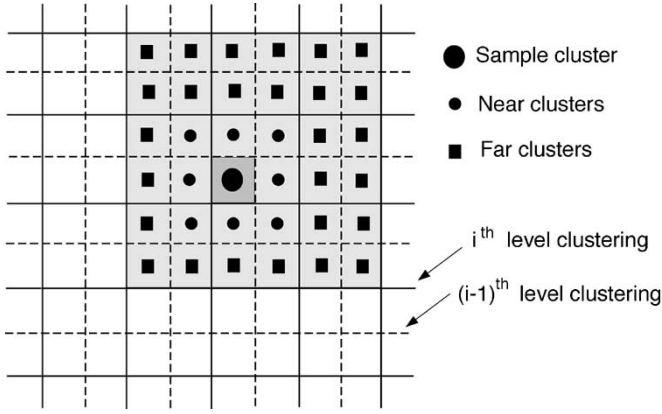


Fig. 3. Two-dimensional multilevel clustering example.

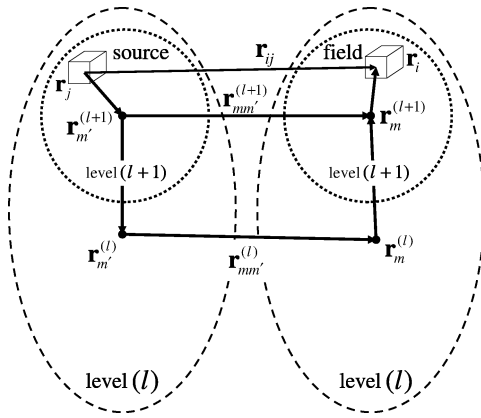


Fig. 4. Two-level FMM construct for a general multilevel case.

we proceed to expand the scalar Green's function as

$$\frac{e^{ikr_{ij}}}{r_{ij}} = \frac{ik}{4\pi} \int d^2\hat{k} e^{i\mathbf{k} \cdot (\mathbf{r}_{im}^{(l+1)} + \mathbf{r}_{mm}^{(l)})} \times \alpha_{L(l)} \left(kr_{mm'}^{(l)}, \hat{k} \cdot \hat{\mathbf{r}}_{mm'}^{(l)} \right) e^{-i\mathbf{k} \cdot (\mathbf{r}_{m'm'}^{(l)} + \mathbf{r}_{jm'}^{(l+1)})} \quad (12)$$

where

$$\alpha_{L(l)} \left(kr_{mm'}^{(l)}, \hat{k} \cdot \hat{\mathbf{r}}_{mm'}^{(l)} \right) = \sum_{q=0}^{L(l)} i^q (2q+1) h_q^{(1)} \left(kr_{mm'}^{(l)} \right) P_q \left(\hat{k} \cdot \hat{\mathbf{r}}_{mm'}^{(l)} \right) \quad (13)$$

is the so called translation operator with the superscripts $(l+1)$ and (l) denoting the grouping levels, (l) being the coarser level and $(l+1)$ the finer. In (13) $h_q^{(1)}$ is the spherical Hankel function of the first kind of order q and P_q is the Legendre polynomial of order q . Hence, the dyadic Green's function can be written as

$$\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j) = \frac{ik}{4\pi} \int d^2\hat{k} [\mathbf{I} - \hat{k}\hat{k}] e^{i\mathbf{k} \cdot \mathbf{r}_{im}^{(l+1)}} e^{i\mathbf{k} \cdot \mathbf{r}_{mm}^{(l)}} \alpha_{L(l)} \left(kr_{mm'}^{(l)}, \hat{k} \cdot \hat{\mathbf{r}}_{mm'}^{(l)} \right) \cdot [\mathbf{I} - \hat{k}\hat{k}] e^{-i\mathbf{k} \cdot \mathbf{r}_{m'm'}^{(l)}} e^{-i\mathbf{k} \cdot \mathbf{r}_{jm'}^{(l+1)}} \quad (14)$$

For the far-zone elements in (4), upon discretizing the spectral integral in (12) using numerical quadrature (see [15]), we obtain

$$Z_{ij} = \langle \mathbf{e}_j(\mathbf{r}), \mathbf{e}_i(\mathbf{r}) \rangle - \frac{ik}{4\pi} \sum_{\hat{k}^{(l)}} w_{\hat{k}^{(l)}} \mathbf{V}_{fim} \left(\hat{k}^{(l+1)} \right) \times e^{i\mathbf{k}^{(l)} \cdot \mathbf{r}_{mm}^{(l)}} \alpha_{L(l)} \left(kr_{mm'}^{(l)}, \hat{k}^{(l)} \cdot \hat{\mathbf{r}}_{mm'}^{(l)} \right) \cdot e^{-i\mathbf{k}^{(l)} \cdot \mathbf{r}_{m'm'}^{(l)}} \mathbf{V}_{sjm'}^* \left(\hat{k}^{(l+1)} \right) \quad (15)$$

where

$$\mathbf{V}_{fim}(\hat{k}) = \int_v dv' e^{i\mathbf{k} \cdot \mathbf{r}_{im}} [\mathbf{I} - \hat{k}\hat{k}] \mathbf{e}_i(\mathbf{r}_i) \quad (16)$$

and

$$\mathbf{V}_{sjm'}(\hat{k}) = \int_v dv' e^{i\mathbf{k} \cdot \mathbf{r}_{jm'}} [\mathbf{I} - \hat{k}\hat{k}] \mathbf{e}_j(\mathbf{r}_j) \quad (17)$$

are the so called signature functions (often referred to as the Fourier transforms of the basis functions). We note here that the integrals in (16) and (17) are evaluated over the hexahedral domains of each basis function and the vectors $\mathbf{r}_i$ and $\mathbf{r}_j$ are the integration variables.

However, since the discrete values of the signature functions $\mathbf{V}_{fim}(\hat{k}^{(l+1)})$ and $\mathbf{V}_{sjm'}^*(\hat{k}^{(l+1)})$ are for the k -space integral at level $(l+1)$, their values for the k -space discretization at level (l) must be computed via interpolation, viz.

$$\mathbf{V}_{fim} \left(\hat{k}^{(l)} \right) = W \left\{ \mathbf{V}_{fim} \left(\hat{k}^{(l+1)} \right) \right\} = \mathbf{W}_{(l),(l+1)} \mathbf{V}_{fim} \left(\hat{k}^{(l+1)} \right) \quad (18)$$

where $\mathbf{W}_{(l),(l+1)}$ refers to the interpolation coefficients.

It is necessary that the multilevel grouping algorithm provide the neighborhood information of all clusters at all levels. When constructing the far-zone clusters for the i th cluster at level $(l+1)$, it is implied that the parents (at level (l)) of clusters i and j (at level $(l+1)$) must be in the near-zone of each other. Otherwise, the interactions of clusters i and j are computed through translations at level (l) .

The matrix-vector product for this two-level grouping can be carried out as follows: The aggregation is done for the source cluster at the finer level $(l+1)$ for each k -direction $\hat{k}^{(l+1)}$. The k -space sampling of the aggregated field signature of the source cluster is then interpolated to compute the signature on different k -space samples at level (l) using (18). The phase center of the aggregated field signature of the source cluster is then shifted to the center of the source cluster at the coarser level (l) by simply multiplying with $e^{-i\mathbf{k}^{(l)} \cdot \mathbf{r}_{m'm'}^{(l)}}$ as in (16). After translating the signature onto the field cluster at this coarse level, the same operations are carried out in the reverse order to compute the final matrix-vector product. Namely, the phase center of the translated field is shifted to the center of the field cluster at the finer level on multiplying by $e^{i\mathbf{k}^{(l)} \cdot \mathbf{r}_{m'm'}^{(l)}}$. This phase shifted field is finally interpolated onto the k -space integration points at the finer level $(l+1)$ using the transpose of the interpolation matrix.

It should be noted that the complexity of the MLFMM relies on the use of a sparse interpolation technique. Since the signature functions are smooth on the unit sphere, a fourth-order

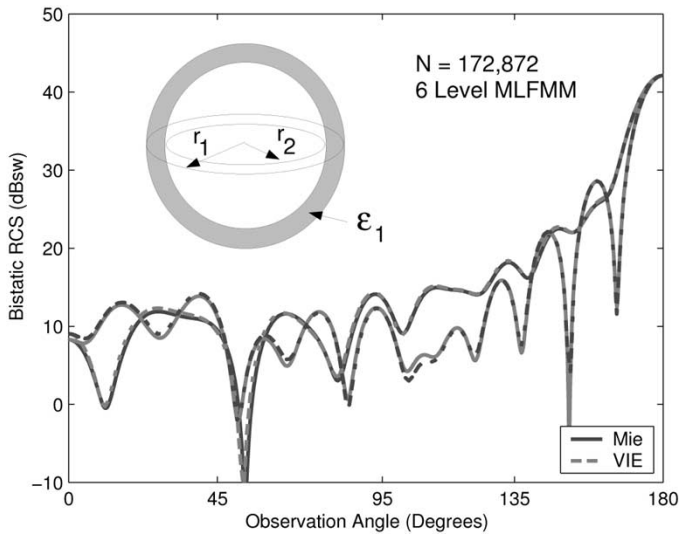


Fig. 5. Bistatic RCS of a 2λ radius dielectric spherical shell ($\epsilon_r = 2.75 + 0.3i$).

piecewise-polynomial interpolation, generating a sparse interpolation matrix, has been adopted in this work.

IV. NUMERICAL RESULTS

A conjugate gradient squared (CGS) solver was used to carry out the numerical solutions given in this section. We note that since the VIE in (1) is a second kind integral equation, good convergence is expected and indeed observed, as reported in Section IV. In all cases, for the number of multipoles used in the MLFMM implementation, the semiempirical formula $L = kD + 2\log(kD + \pi)$ was adopted [15], where D is the diameter of the groups.

The first dielectric geometry considered is a spherical dielectric shell of outer radius $2\lambda_0$ and a thickness $0.2\lambda_0$ having $\epsilon_r = 2.75 + 0.3i$. Fig. 5 shows the computed bistatic RCS as compared to the analytic data obtained from the Mie series. The excellent agreement (within less than 2% rms error) between the computed and analytic results over a 40-dB dynamic range can be attributed to the higher order geometry modeling using curved elements and conformal basis functions. The geometry was modeled using 57 624 curvilinear hexahedra and resulted in 172 872 unknowns ($\Delta \approx 0.07\lambda_0$). The MLFMM used six levels to converge to a residual of 0.01 within only 32 iterations. The number of multipoles used at each level were 4,5,7,10,16,28.

As a second example, we considered the scattering by a dielectric sphere coated with a dielectric shell. The spherical core is of radius $0.9\lambda_0$ and has a dielectric constant of $\epsilon_r = 1.75 + 0.3i$, whereas the encapsulating shell is of thickness $0.1\lambda_0$ and has a dielectric constant of $\epsilon_r = 1.25 + 1.25i$. This geometry was modeled using 23 125 elements (69 375 unknowns). The simulation was done using five MLFMM levels, and convergence was achieved in 231 CGS iterations. Also, the number of multipoles for this example were 4,5,7,10,16. Again, we observe the remarkably fast convergence rate, and accuracy (within less than 2.5% rms error, as seen in Fig. 6) using nominal sampling.

Our final example demonstrates the accuracy of the method for high contrast dielectric materials. The geometry is a two-layer material coated sphere with the permittivity of the sphere's

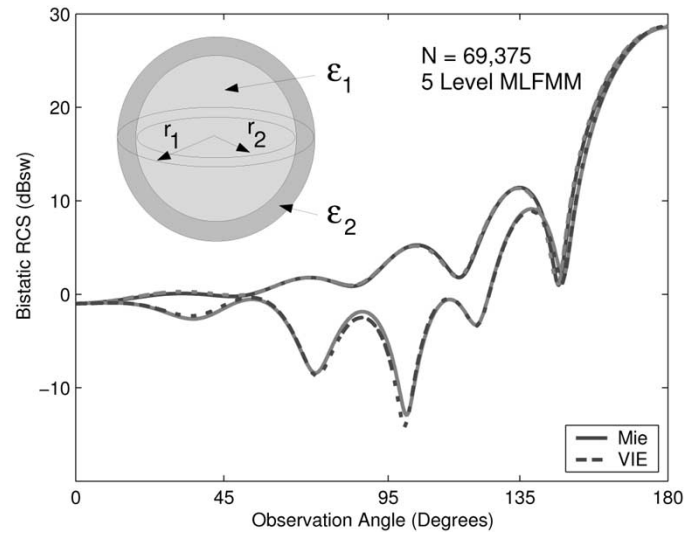


Fig. 6. Bistatic RCS of a dielectric coated dielectric sphere ($\epsilon_1 = 1.75 + 0.3i$, $\epsilon_2 = 1.25 + 1.25i$).

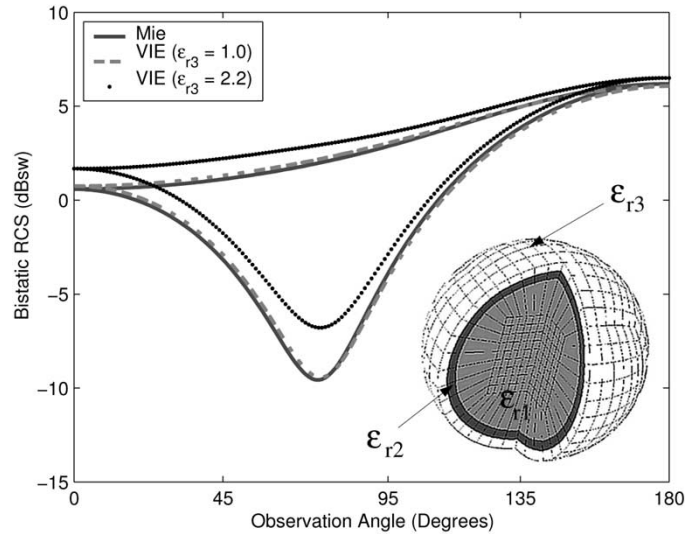


Fig. 7. Bistatic RCS of a two-layer dielectric coated sphere.

core of the sphere being $\epsilon_{r1} = 1.25$ up to a radius of $0.18\lambda_0$, λ_0 being the free-space wavelength. For the first layer coating, the thickness is $0.02\lambda_0$ and $\epsilon_{r2} = 15.0$ representing a contrast of 12 to 1 at the boundary between the layer and the sphere's core. The outermost layer also has a thickness of $0.02\lambda_0$ and a permittivity ϵ_{r3} . Considering the wavelength in the high permittivity shell ($\epsilon_{r2} = 15.0$), the volumetric mesh for this region is constructed by specifying the maximum edge length to be around $(\lambda_0/10)/\sqrt{\epsilon_{r2}} \approx 0.0258\lambda_0$. The rest of the volume mesh is necessarily constructed to be conformal with the mesh in this region. Consequently, even though the electrical size of the geometry is only $0.4\lambda_0$, the resulting matrix size is 21 567. By comparison, a PEC sphere of the same electrical size would require at most only a few hundred unknowns. This example clearly demonstrates the need of using fast integral methods for volumetric scatterers. We considered two separate cases for the outermost layer. In one case, this layer was assumed to be a dummy air layer ($\epsilon_{r3} = 1.0$). The bistatic RCS for $\epsilon_{r3} = 1.0$ is given in Fig. 7 and is seen to be in very good agreement with

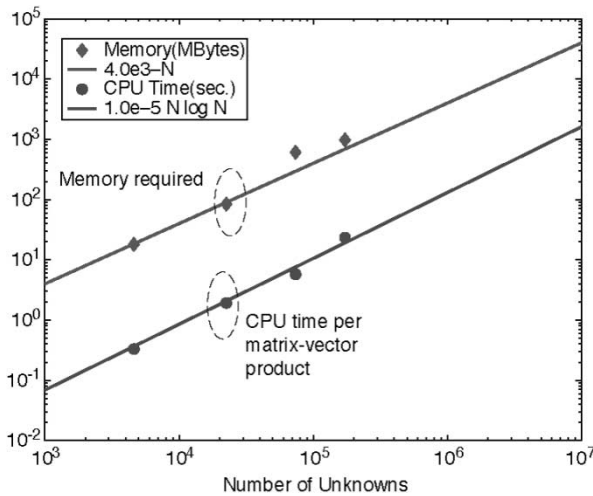


Fig. 8. CPU and memory performance of volumetric MLFMM.

the analytical Mie series result. Convergence was achieved in 24 CGS iterations and the rms error between the numerical RCS results and the Mie series data is less than 2.7%. For the other case, the outermost layer was assumed to have $\epsilon_{r3} = 2.2$ and the bistatic RCS result is again given in Fig. 7. The CGS solver required 32 iterations. It is interesting to observe that a change of ϵ_{r3} from 1.0 to 2.2 ($0.02\lambda_0$ thick) leads to as much as 4 dB difference between the curves at their low values. We note here that although we did not plot the RCS data, for the case of lossy sphere core ($\epsilon_{r1} = 1.25 + 1.25i$, $\epsilon_{r2} = 15.0$, $\epsilon_{r3} = 2.2$) convergence was achieved in 42 iterations.

Based on the above examples, we computed the CPU and memory requirements of the MLFMM for VIEs as functions of unknowns. Fig. 8 shows the CPU and memory curves of the proposed MLFMM VIE formulation. The actual CPU times refer to a 1 GHz Pentium III personal computer. It is not surprising that as compared to an FE-BI solver, the VIE solver converges much faster, and is also free of internal resonance issues. However, since the generation of the matrix entries Z_{ij} involves double volume integrals, more time is required to fill the system as compared to the FE-BI approach. Also, the system matrix for the VIE formulation is completely dense, whereas the FE-BI formulation leads to partly sparse and partly dense matrix. The inherent error propagation associated with the finite element method may likely render the FE-BI solution less attractive for large-scale simulations.

A final important point to mention is that the MLFMM becomes more efficient as the number of volume unknowns increase. Hence, the VIE is also attractive for high contrast dielectrics where higher sampling and more unknowns are required. These comments certainly require further examination to develop comparative results for a variety of geometries and compositions, and thus an appropriate subject of a future paper.

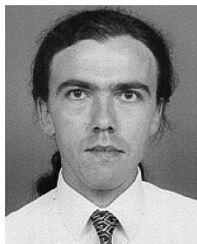
V. CONCLUSION

We presented a MLFMM implementation of the VIE formulation for inhomogeneous dielectric structures modeled using curvilinear hexahedral elements and piecewise-constant electric field basis functions. The basis functions presented here are a generalization of those given in [2] and by virtue of being

piecewise-constant, they provide case for domain decomposition and parallelization. The efficiency and the accuracy of the curvilinear hexahedral elements were demonstrated with the excellent convergence rate of the resulting VIE systems (convergence in 32 iterations for a 172 872 unknown system). These advantages are expected to be even more pronounced for larger matrix systems as is the case for higher contrast dielectrics and larger volumes. We also noted that the proposed MLFMM VIE has a better convergence rate than the standard FE-BI and is free from internal resonances in addition to having lower error propagation.

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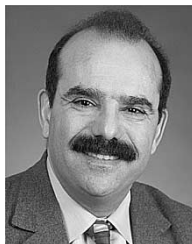
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