

A Fast Algorithm Based on Volume Integral Equation for Analysis of Arbitrarily Shaped Dielectric Radomes

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Abstract—The volume integral equation (VIE) combined with the multilevel fast multipole algorithm (MLFMA) is applied to analyze antenna radiation in the presence of dielectric radomes. In solving the VIE, the radome is modeled by small volume cells of tetrahedron or hexahedron shape so that three-dimensional (3-D) complex radomes can be modeled accurately. When the induced volume current is determined by the method of moments, the total radiation is calculated by adding the induced current radiation to that of the antenna in the absence of the radome. The application of MLFMA reduced the computational complexity to a lower order and hence electrically large-sized radomes are analyzed. Numerical results are compared with the analytical solution for spherical shells with dipole array as excitations. The radiation patterns of dipole arrays in the presence of ogive, cone, and hemisphere radomes are also presented.

Index Terms—Antenna radome, electromagnetic scattering, fast algorithm, integral equation.

I. INTRODUCTION

THE PRESENCE of radome will generally affect the radiation characteristics of antennas due to the radiation of induced current on the radome. An important problem associated with the presence of radome is the antenna beam pointing error caused by radome. Hence, accurately analyzing beam deformation by radome can help design engineers to compensate phase shift in the array elements to correct the errors. Techniques for analyzing the antenna pattern deformation are usually high frequency methods (such as GTD and PO [1]–[4]) because the size of a realistic radome is often electrically large. A fundamental assumption for the high frequency methods is that the radome surface is large and smooth, and has electrically large radius of curvature. This assumption is valid for the majority portion of any realistic radomes. However for some radomes that contain surface discontinuities such as the tip portion of an ogive radome, the solutions predicted by a high frequency method alone are usually not accurate. On the other hand, the full-wave methods such as the finite-element method (FEM) and the method of moments (MoM) provide much higher solution accuracy compared to that by the high-frequency methods. However, these full-wave methods apply to electrically small-

sized radomes because these methods have either high computational complexity (e.g., MoM), or the number of unknowns increases rapidly with the increase of the radome size (e.g., FEM needs to mesh the volume region including the air-filled interior of the radome).

This paper presents a method that applies the multilevel fast multipole algorithm to solve the volume integral equations (VIE) so that radiation and scattering problems involving electrically large object can be solved. The method is based on the volume integral equation approach, and the increased solution capability is due to the application of the multilevel-fast multipole algorithm. Though the VIE has been studied previously and applied to a number of problems by several investigators [5]–[7], the applications of these studies are limited to electrically small dielectrics due to the high computational complexity in solving the resultant matrix equation. In order to increase the object size that can be solved by the VIE, one way is to reduce the computational complexity. As shown in the next section, the formulation of the VIE used the Green's function for the background medium, and the electrical field VIE is similar to that of the electric field surface integral equation (SIE) (except for the integration domain), hence, applying MLFMA is similar to the implementation of the algorithm to SIE. The similarity is that for the same group size, the operation counts of well-separated interactions for VIE case and SIE case are essentially the same. An obvious difference is the operation and memory for the near-neighbor interactions. In general, the number of elements in a group of volume cells is larger than that in a group of surface patches. This will be discussed further in Section III. It should be pointed out that a similar approach based on the SIE for dielectrics is also possible. In general, the SIE produces less number of unknowns for homogeneous dielectrics. However the numbers of unknowns are of the same order for thin and layered medium such as those in the radomes. In addition, the VIE can be applied to inhomogeneous dielectric.

As a remark, in general the antenna's current distribution will also be affected when it is placed inside a radome [20]. Though the method presented in this paper can be extended to calculate the antenna's current distribution after the radome is introduced, this paper is limited to a special case where the excitation is a short dipole or dipole array. Hence, the primary field is not affected by the radome. The following sections will outline the formulation of the problem and presents numerical simulation results. The problem is formed in frequency domain and the time factor of $\exp\{-i\omega t\}$ is suppressed.

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II. FORMULATION

A. The VIE

Consider a dielectric shell of arbitrary shape resides in a homogeneous background medium of infinite extent with ε_b , μ_b as permittivity and permeability parameters, as shown in Fig. 1. The dielectric has position dependent permittivity $\varepsilon(\mathbf{r})$. To simplify the formulation, this paper assumes that the permeability of the material is a constant $\mu = \mu_b$. The source of excitation is a dipole or dipole array located at a fixed position in the background medium (it can be inside as well as outside the radome). It is expected that the presence of the dielectric radome will in general affect the radiation pattern of the antenna. By equivalence principle, the total radiation is the sum of the antenna's primary radiation and the radiation of the induced volume current of the dielectric in the background medium, i.e.,

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + i\omega\mu_b \int_V G(\mathbf{r}, \mathbf{r}') \mathbf{J}_V(\mathbf{r}') dV' - (1/i\omega\varepsilon_b) \nabla \int_V G(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_V(\mathbf{r}') dV' \quad (1)$$

where V is the radome region, $\mathbf{E}^i$ is the antenna's primary radiation in the absence of the radome, $\mathbf{E}$ is the total electric field, $\mathbf{J}_V$ is the induced volume current in the radome, $G(\mathbf{r}, \mathbf{r}') = \exp\{ik_b R\}/(4\pi R)$ is the three-dimensional (3-D) scalar Green's function for the background media, $R = |\mathbf{r} - \mathbf{r}'|$ is the distance from a source point $\mathbf{r}'$ to a field point $\mathbf{r}$, and $k_b = \omega\sqrt{\varepsilon_b\mu_b}$ is the wavenumber of the background medium. By writing $\varepsilon(\mathbf{r}) = \varepsilon_b + [\varepsilon(\mathbf{r}) - \varepsilon_b]$ in Maxwell's equations, one identifies that the volume current is related to the total electric field by

$$\mathbf{J}_V(\mathbf{r}) = -i\omega[\varepsilon(\mathbf{r}) - \varepsilon_b]\mathbf{E}(\mathbf{r}) = [\varepsilon_b/\varepsilon(\mathbf{r}) - 1]i\omega\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}). \quad (2)$$

In (1), the two integrals on the right-hand side (RHS) represent the radiation from the induced volume current and the corresponding charge. It is this radiation that affects the radiation pattern of the antenna. In other words, (1) allows us to calculate the true radiation pattern for the antenna-radome system if the induced current is determined. It should be pointed out that the position vector $\mathbf{r}$ in (1) can be at any location in space. Hence, if the field point $\mathbf{r}$ is restricted in the radome region V , then an integral equation for $\mathbf{E}$ or $\mathbf{J}_V$ is formed from which either $\mathbf{E}$ or $\mathbf{J}_V$ can be solved. This integral equation is called the VIE because the unknown function is varying in a volume region.

B. Discretization of the VIE

The MoM has been applied to solve the VIE [8]–[12]. In applying a subdomain MoMs, one needs to subdivide the dielectric region into small volume cells. Previous studies used cubic model [8], [10], [11], as well as tetrahedron models [9], [12]. The conjugate gradient fast Fourier transform (CGFFT) [10], [11] algorithm can be easily implemented using the structured cubic model. However, it is not accurate in modeling structures with curved boundaries. The geometric modeling error results in large solution error if the dielectric contrast is high. On the other hand, the tetrahedron shape and hexahedron shape are suitable for modeling arbitrarily shaped dielectric structures. Hence both

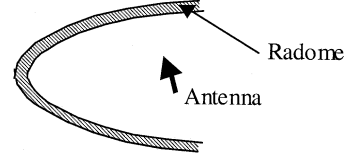


Fig. 1. The cross view of the antenna-radome configuration.

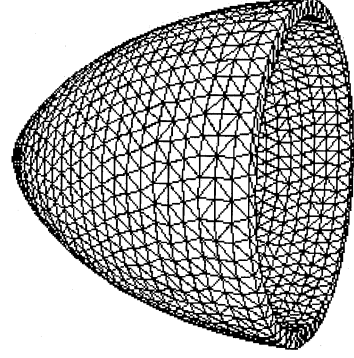


Fig. 2. The tetrahedron mesh of an ogive-shaped radome.

of the two shapes are considered in the implementation of the algorithm in this paper. An example of the tetrahedron mesh for an ogive radome is shown in Fig. 2.

To solve the integral equation, the unknown vector $i\omega\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$ is first represented by a set of N basis functions $\mathbf{f}_j$, $j = 1, 2, \dots, N$, such that

$$\mathbf{J}_V(\mathbf{r}) = [\varepsilon_b/\varepsilon(\mathbf{r}) - 1]i\omega\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) = \chi(\mathbf{r}) \sum_{j=1}^N a_j \mathbf{f}_j(\mathbf{r}) \quad (3)$$

where $\chi(\mathbf{r}) = \varepsilon_b/\varepsilon(\mathbf{r}) - 1$. The basis function is defined differently for different cell shapes. In general a basis function $\mathbf{f}_j$ is defined on two adjacent cells, Ω_j^+ and Ω_j^- , that share the same face S_j . This paper uses the extended RWG function for tetrahedron mesh [9]. Following the MoM steps as described in [9], [11]–[13], the integral equation (1) can be converted into a matrix equation that can be solved for the expansion coefficients a_j s. The matrix elements are given by

$$Z_{ij} = - \int_{\Omega_i} \frac{\mathbf{f}_i \cdot \mathbf{f}_j}{i\omega\varepsilon(\mathbf{r})} dV + \int_{\Omega_i} \mathbf{f}_i \cdot (i\omega\mathbf{A}_j - \nabla\Phi_j) dV \quad (4)$$

where

$$\mathbf{A}_j(\mathbf{r}) = \mu_b \int_{\Omega_j} G(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{f}_j(\mathbf{r}') dV' \quad (5)$$

$$\Phi_j(\mathbf{r}) = (1/i\omega\varepsilon_b) \int_{\Omega_j} G(\mathbf{r}, \mathbf{r}') \nabla' \cdot [\chi(\mathbf{r}') \mathbf{f}_j(\mathbf{r}')] dV'. \quad (6)$$

While the discretization of the VIE into matrix equation is a straight-forward MoM approach, two issues specific to VIE needs to be pointed out.

The first issue is the selection of unknown function. Using the induced volume current $\mathbf{J}_V$ as the unknown function has an advantage that the integral kernel will not depend on the material parameter. In this paper, however, we used $i\omega\varepsilon\mathbf{E}$ as the unknown function [the volume current is related to this function

via (3)]. This is to ensure the continuity of the normal component of electric flux density $\mathbf{D}$, since the roof-top basis functions automatically satisfy this condition. Because of this selection, the vector $\mathbf{V}_{snj}(\hat{k})$ [see (14)] is no longer related to $\mathbf{V}_{fmi}(\hat{k})$ via a complex conjugate operation. Moreover, if the permittivities in the two adjacent cells are different, and hence χ are different, then there will be a discontinuity in the normal component of the volume current $\mathbf{J}_V$ since $\mathbf{J}_V(\mathbf{r}) = \chi(\mathbf{r}) i\omega\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$, and the normal component of $i\omega\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$ is continuous across the dielectric interface. The continuity condition requires that a charge distribution be introduced to account for the discontinuity in $\mathbf{J}_V$. It can be shown [16], [9] that the induced surface charge is proportional to $(\chi_j^+ - \chi_j^-)$ the difference in the permittivities of the two volume cells. This does not lead to additional number of degrees of freedom (i.e., the charge is associated with the current). In fact, for piece-wise continuous χ among the cells, the integral associated to the electric potential in (6) can be written as

$$\int_{V_j^\pm} G(\mathbf{r}, \mathbf{r}') \cdot \nabla' (\chi_j^\pm \mathbf{f}_j) dV' = \int_{V_j^\pm} \chi_j^\pm G(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{f}_j dV' + \int_S G(\mathbf{r}, \mathbf{r}') [\chi_j^+ - \chi_j^-] \mathbf{f}_j \cdot d\mathbf{S}'. \quad (7)$$

In (7), the second integral on the RHS is over the common face of the two adjacent cells. As is expected, if the two adjacent cells have the same permittivity, leading to the same χ for both cells, then there will be no net charge on the interface. The above result is also obtained by Schaubert, Wilton, and Glisson [9] for the tetrahedron cells.

On the other hand, if the induced volume current $\mathbf{J}_V$ is chosen as basis functions, then auxiliary charge must be introduced on the interface between two volume cells to satisfy boundary condition. The author is not aware of any published papers that deal with this additional charge. But this is not a problem if pulse basis functions are used to expand the unknown function $\mathbf{J}_V$. Most of the earlier papers on VIE used pulse basis function and hence it is natural to use $\mathbf{J}_V$ as the unknown function in those studies.

The other issue is the mesh truncation. In general, the vector $i\omega\epsilon\mathbf{E}$ can be nonzero in the entire background media. Hence it is necessary to extend the mesh into the whole space. However, by (2), only the basis functions defined on cells belonging to the dielectric material region are used to represent the induced current $\mathbf{J}_V$ (because $\chi(\mathbf{r}) = 0$ if $\mathbf{r}$ is outside the dielectric region). Therefore the mesh should be truncated such that only those volume cells that are either inside the dielectric region or have direct contact with the dielectric volume are retained. The cell that does not belong to the dielectric region but has direct contact with a cell inside the dielectric region is called an auxiliary cell, as shown in Fig. 3. The existence of auxiliary cells introduces several difficulties: 1) it extends the source integration in (5) and (6), and the field integration in (4) to beyond the true dielectric object; 2) it increases the computation time because of the extra integral regions; 3) it needs extra mesh generating task which may not always be possible. As a result, in practical implementation of the algorithm, the auxiliary cells are eliminated

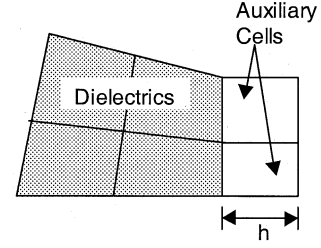


Fig. 3. Cells in dielectrics (shaded) and auxiliary cells (unshaded). The auxiliary cells are introduced to terminate the mesh (the dimension h is arbitrary and has been taken to be zero in the numerical implementation).

by, for example, taking the limit of $h \rightarrow 0$. This is possible because there is no restriction on the size of the auxiliary cell as long as it shares a face with a cell in the dielectric region. As a result, a boundary basis function that is originally associated with a dielectric cell and an auxiliary cell now is defined on the dielectric cell only. This basis function is called a “half” basis function since it is associated with only one cell as compared to a regular (or full) basis function that is defined on two cells.

C. Application of MLFMA

If the matrix equation is solved by an iterative method such as the conjugate gradient (CG) method, the computational complexity for performing a matrix vector multiplication is $O(N^2)$ for an N -unknown problem. Apparently, because of this complexity, both memory requirement and CPU time can become prohibitively high when N is large. Hence direct solution by an iterative solver for the set of linear equations is not possible for electrically large radomes. This is the most important reason that previous studies of dielectric scattering problems by MoM are limited to small-sized structures. However the above-mentioned complexity for a matrix vector multiplication can be reduced to a lower order if the multilevel fast multipole algorithm (MLFMA) is employed. The MLFMA [16]–[19] is the multilevel implementation of the fast multipole method (FMM) [14], [15] that was first used to solve wave scattering problems by Rokhlin. The principle of FMM is based on the addition theorem for spherical harmonics. The FMM and MLFMA have been applied to solving electromagnetic wave scattering problems for conducting objects. If the SIE for conducting objects and the VIE for dielectrics are compared, one finds that the two equations are very similar except for the domain of integration. As a result, MLFMA can also be applied to speed up the solution of VIE with some modifications. The following presents the process to implement FMM with focus on the application to solving VIE. For a given testing function, all the basis functions can be classified into two categories based on the distance between the testing function and the basis function: one as near-neighbor elements (NN) and one as well-separated, or far-field (FF) elements. As a result, a matrix-vector multiplication is written as a sum of two terms,

$$\sum_j Z_{ij} a_j = \sum_{j \in NN} Z_{ij} a_j + \sum_{j \in FF} Z_{ij} a_j. \quad (8)$$

The near-neighbor elements are calculated directly using (4) and are stored in a sparse matrix. For far-field matrix elements, an

alternative form of the integral operator is used, i.e., the form based on dyadic Green's function

$$Z_{ij} = i\omega\mu_b \int_{V_i} \int_{V_j} \mathbf{f}_i(\mathbf{r}) \cdot \left(\bar{\mathbf{I}} + \frac{1}{k_b^2} \nabla \nabla \right) \cdot G(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \mathbf{f}_j(\mathbf{r}') dV' dV. \quad (9)$$

Assuming the dielectric region is subdivided into M small rectangular boxes. The basis functions belonging to one box form a group. Let basis function $\mathbf{f}_j$ belong to group- n centered at $\vec{r}_n$, and testing function $\mathbf{f}_i$ belong to group- m centered at $\vec{r}_m$. Then, the source to field vector $\vec{R} = \vec{r} - \vec{r}'$ can be written as $\mathbf{r} - \mathbf{r}' = (\mathbf{r} - \mathbf{r}_m) + (\mathbf{r}_m - \mathbf{r}_n) + (\mathbf{r}_n - \mathbf{r}') = \mathbf{d} + \mathbf{r}_{mn}$, where $\mathbf{d} = (\mathbf{r} - \mathbf{r}_m) + (\mathbf{r}_n - \mathbf{r}')$, and $\mathbf{r}_{mn} = \mathbf{r}_m - \mathbf{r}_n$. For $|\mathbf{r}_{mn}| > |\mathbf{d}|$, the addition theorem can be used to expand the scalar Green's function $G(\mathbf{r}, \mathbf{r}')$ into a multipole expression [15]

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{ik_b|\mathbf{r}_{mn}+\mathbf{d}|}}{4\pi|\mathbf{r}_{mn}+\mathbf{d}|} = \frac{ik_b}{(4\pi)^2} \int d^2\hat{k} e^{ik_b\hat{k}\cdot\mathbf{d}} T_{mn}(\hat{k}) \quad (10)$$

where

$$T_{mn}(\hat{k}) = \sum_{l=0}^{\infty} i^l (2l+1) h_l^{(1)}(k_b r_{mn}) P_l(\hat{r}_{mn} \cdot \hat{k}) \quad (11)$$

$h_l^{(1)}(\cdot)$ is the spherical Hankel function of the first kind, and $P_l(\cdot)$ is the Legendre function of order l . In practical implementations, the infinite series in (11) is approximated by L terms. The selection of L depends on the accuracy desired and reference [15] gives some guidelines on how to choose this truncation parameter. The integral in (10) is over a unit sphere and it can be numerically evaluated with Q integral nodes ($Q \approx k_b d$). Using (10), the dyadic Green's function can be expressed as

$$i\omega\mu_b \left(\bar{\mathbf{I}} + \frac{1}{k_b^2} \nabla \nabla \right) G(\mathbf{r}, \mathbf{r}') = \frac{-\eta_b k_b^2}{(4\pi)^2} \int d^2\hat{k} (\bar{\mathbf{I}} - \hat{k}\hat{k}) e^{ik_b\hat{k}\cdot\mathbf{d}} T_{mn}(\hat{k}). \quad (12)$$

Using the identity of $(\bar{\mathbf{I}} - \hat{k}\hat{k}) = (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k})$, and noting $\mathbf{d} = (\mathbf{r} - \mathbf{r}_m) + (\mathbf{r}_n - \mathbf{r}')$, we have

$$\begin{aligned} \mathbf{f}_i(\mathbf{r}) \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k}) e^{ik_b\hat{k}\cdot\mathbf{d}} \cdot \chi \mathbf{f}_j(\mathbf{r}') \\ = \underbrace{\mathbf{f}_i(\mathbf{r}) \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k}) e^{ik_b\hat{k}\cdot(\mathbf{r}-\mathbf{r}_m)}}_{\text{Depends on } \mathbf{r} \text{ and } \hat{k}} \\ \cdot \underbrace{\chi \mathbf{f}_j(\mathbf{r}') \cdot (\bar{\mathbf{I}} - \hat{k}\hat{k}) e^{ik_b\hat{k}\cdot(\mathbf{r}_n-\mathbf{r}')}}_{\text{Depends on } \mathbf{r}' \text{ and } \hat{k}} \end{aligned}$$

As a result, the dyadic Green's function has been written as summation, in which each term consists of two factors, one depends on $\mathbf{r}$, and the other depends on $\mathbf{r}'$. Substituting (12) into (9), the impedance matrix element corresponding to well-separated testing and basis functions can be written as

$$Z_{ij} = \frac{-\eta_b k_b^2}{(4\pi)^2} \int d^2\hat{k} \mathbf{V}_{fmi}(\hat{k}) T_{mn}(\hat{k}) \cdot \mathbf{V}_{snj}(\hat{k}) \quad (13)$$

where the vectors $\mathbf{V}_{fmi}$ and $\mathbf{V}_{snj}$ are given by

$$\begin{aligned} \mathbf{V}_{fmi}(\hat{k}) &= \int_{V_i} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \mathbf{f}_i(\mathbf{r}) e^{ik_b\hat{k}\cdot(\mathbf{r}-\mathbf{r}_m)} dV, \\ \mathbf{V}_{snj}(\hat{k}) &= \int_{V_j} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \mathbf{f}_j(\mathbf{r}') \chi(\mathbf{r}') e^{-ik_b\hat{k}\cdot(\mathbf{r}'-\mathbf{r}_n)} dV'. \end{aligned} \quad (14)$$

$$(15)$$

Though the form of the far-field matrix element in (13) is identical to that of the SIE case, there are differences in the implementations for VIE and SIE cases. First of all, in the case of SIE, $\mathbf{V}_{fmi}$ and $\mathbf{V}_{snj}$ are complex conjugates to each other if the Galerkin's test is applied. This allows one to store only one of $\mathbf{V}_{fmi}$ or $\mathbf{V}_{snj}$. However, with material present, as indicated by the χ -factor in (15), these two vectors are no longer related and storage spaces must be allocated for both of the two vectors. Second, the actual operations for aggregation and disaggregation processes may be different for SIE and VIE. As an example, let us consider the CPU time needed for the far-field interactions for three pairs of structures: a) a PEC sphere and a dielectric shell of 1 m in radius; b) a PEC sphere and a dielectric shell of 2 m radius; and c) a radome-shaped PEC surface and a dielectric shell of the same shape. The detailed parameters and mesh data are shown in Table I. The comparisons of CPU times as functions of group sizes are shown in Fig. 4. It can be seen that the group sizes for minimum number of floating point operations are different for SIE and VIE cases. This can be explained by the fact that for the same group size, the number of basis functions in VIE case is generally larger than that for the SIE case.

For the shell structured dielectric objects (such as the dielectric radome), the number of basis functions within a group is proportional to the group size, N/M (N is the total number of basis functions). Hence the number of floating point operations and the memory needed to evaluate a matrix-vector multiplication can be shown to be $O(N^{1.5})$. If the above algorithm is implemented in multilevel, the operation count and memory requirement will be reduced to $O(N \log N)$. The reduced computational complexity made it possible for this algorithm to analyze electrically large dielectric structures. The next section shows some numerical results for performance validation of the algorithm and demonstration of the applications.

III. NUMERICAL RESULTS

The MLFMA algorithm for solving the VIE has been implemented using both tetrahedron and hexahedron meshes. In this section, some numerical examples are presented to validate the algorithm as well as to demonstrate the application of the algorithm for analyzing the radiation patterns of dipoles in radomes. As validation, we compare the radiation patterns calculated by this algorithm with analytical solution and SIE solution. In all the examples, the radiation patterns are normalized by the value in the maximum radiation direction.

The first example considers a dipole array inside a spherical dielectric shell, for which the analytical solution is available. The shell's inner and outer radii are 2.4, and 2.47 m, respectively, and the dielectric permittivity is $\epsilon_r = 2 + 1i$. A linear

TABLE I
THE GEOMETRY AND MESH PARAMETERS FOR THREE SURFACE AND THREE SHELL STRUCTURES USED TO SIMULATE CPU TIME OF FAR-FIELD
INTERACTION FOR A TWO-LEVEL MLFMA

	Geometry Description	Mesh parameters	
		# Patch/Cell	# Unknowns
Sphere 1	Surface: Radius=1m	1350	2700
	Shell: Radius=1m, thickness=0.1m	1350	5400
Sphere 2	Surface: Radius = 2m	5400	10800
	Shell: Radius= 2m Thickness=0.1m	5400	21600
Radome	Surface: Parabolic, base radius=0.2m, Height=0.2m	7500	14900
	Save shape as surface, thickness=0.06m	7500	30100

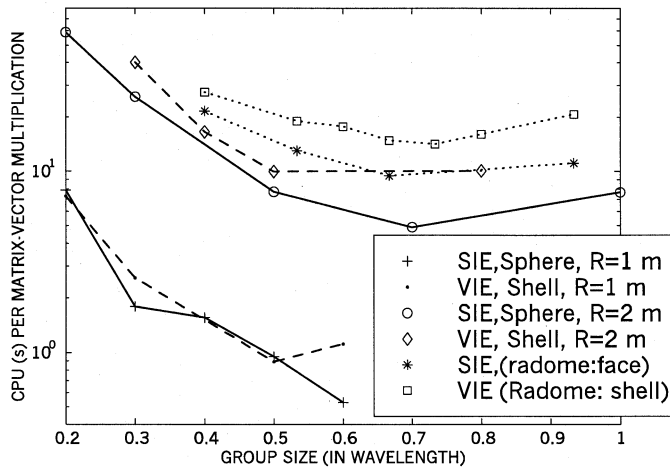


Fig. 4. The CPU time per matrix-vector multiplication (far-field interaction only) as a function of group size.

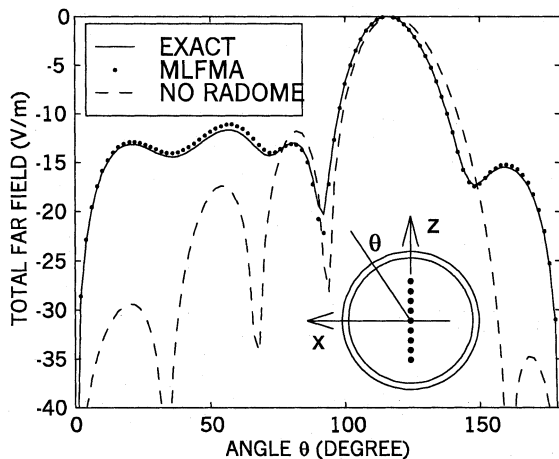


Fig. 5. The radiation pattern of a dipole array in a spherical shell radome calculated by (a) MLFMA and (b) the Mie-series. The dipole elements are uniformly located on z -axis from $z = -1.5$ m to $z = 1.5$ m (as shown by the solid dots inside the circular rings in the figure). The receiver angle is counted from z -axis. The array's pattern in the absence of the radome is also shown as a reference.

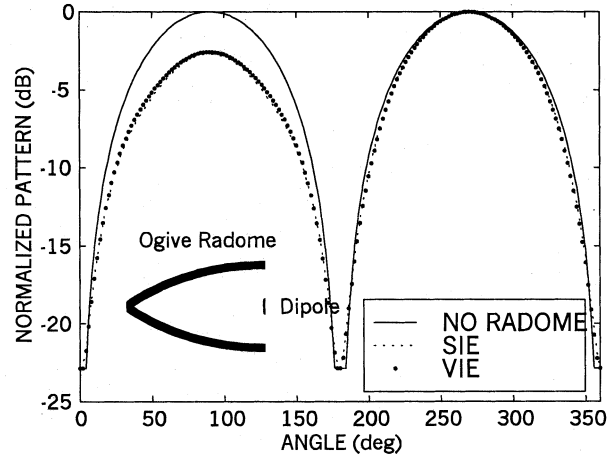


Fig. 6. The comparison of the normalized radiation pattern of a z -directed short dipole in the presence of an ogive dielectric radome calculated by the SIE method and VIE method. The axis of the radome is on x axis and the base is on y - z plane. The horizontal axis of the plot is angle θ . As a reference, the dipole's pattern without the radome is also plotted.

array of 9 short dipoles with equal amplitude and $\pi/4$ progressive phase is placed in the shell (they are uniformly distributed on z -axis from $z = -1.5$ m to $z = 1.5$ m, and are all oriented on z -axis). The radiation pattern of the dipole array at 200 MHz is calculated and shown in Fig. 5. It can be seen that the VIE solution (marked by MLFMA) agrees well with the analytical solution. As reference, the array's radiation in the absence of the shell is also plotted so that the deformation of the pattern due to the presence of the radome can be observed.

To further validate the MLFMA accelerated VIE algorithm for nonspherical radomes, an ogive radome is considered in the next example. The ogive radome is a body of revolution and has base radius of 0.8 m, height of 1.75 m, and thickness of 0.07 m. The dielectric permittivity is $2 + 1j$. The radiation of a dipole at 200 MHz in the radome is plotted in Fig. 6. Also shown in the figure are the result from the SIE and the result of the dipole in free-space. It is seen that the results of VIE and SIE agree well.

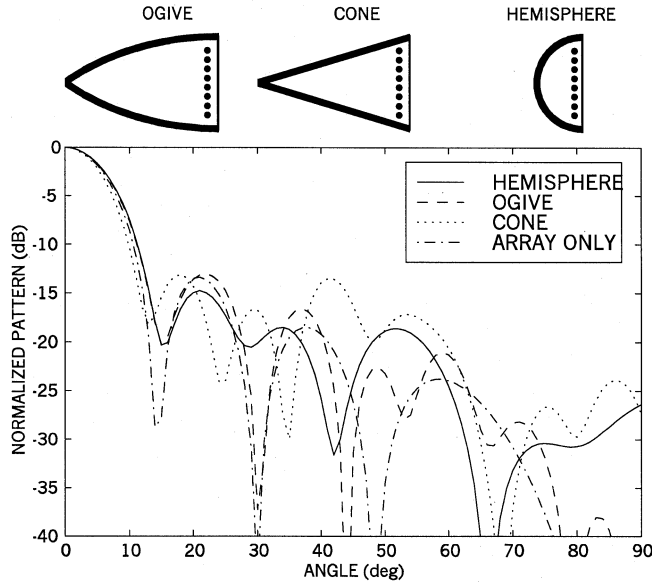


Fig. 7. The comparison of the normalized radiation of a dipole array in the presence of three types of radome shapes. (a) Ogive shape (dash line). (b) Straight cone shape (dotted line). (c) Hemisphere shape (dots). The radiation of the same dipole array in free-space is also plotted for reference. (The cross view of the radome profiles are shown on top of the plot, in which the small solid dots stand for the dipole elements. Dipoles are oriented in z -direction.)

In order to compare the interference effects of different shapes of radome to antenna radiation pattern, the next example shows the calculated results for a dipole array in the presence of three types of radomes: a) a tangent ogive radome with height $6.67\lambda_0$ and base diameter $5.33\lambda_0$ (modeled by 76 914 tetrahedrons); b) a straight cone radome with the same dimension as the ogive (modeled by 55 308 tetrahedrons); c) a hemisphere radome of base diameter $5.33\lambda_0$ (modeled by 45 000 tetrahedrons). The thickness and the permittivities of all three radomes are the same: $t = 0.008\lambda_0$, $\epsilon_r = 2 + 1i$. An 8-element dipole array is placed a quarter wavelength above a conducting plate inside the radome (the plate is positioned at the radome base). The dipole orientations and array axis are in the same direction (perpendicular to the radome axis). The calculated radiation patterns for $\phi = 0$ and θ from 0 to 90° are shown in Fig. 7. As a reference, the radiation pattern of the dipole array in free-space is also plotted in the figure. It can be seen from this figure that the pattern's beam-width in the ogive case and that in the hemisphere cases are about the same, and both are wider than that in the case of cone shape. This is due to the curved surface in the radome. The curved surfaces result in different phase delay for the rays in different direction, and hence they have the larger phase disturbance to the rays compared to that by the straight/flat structures. It is interesting to note that the first side-lobe in the case of cone-shaped radome is only slightly higher than that of the free-space case.

In many realistic applications, the open ends of the dielectric radomes are often terminated by conducting surfaces. To simulate the dipoles' radiation in a radome that is terminated by a conducting plate, we consider the cone-shaped radome as described in the earlier example. Since the radome base diameter is $5.33\lambda_0$, a conducting plate of the same diameter is positioned at the radome base to close the opening. A dipole array

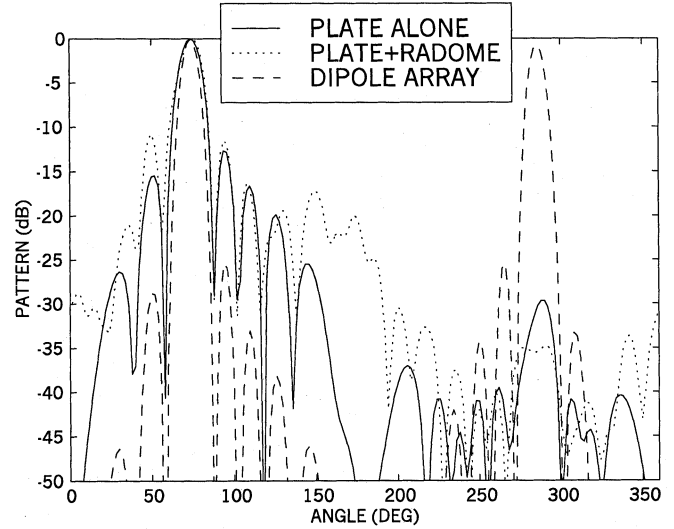


Fig. 8. The radiation pattern of a dipole array in free-space (dash line), over a conducting plate (solid line), and inside radome closed by a conducting plate (dotted line).

of 8-element with equal amplitude and equal phase is positioned a quarter wavelength above the plate and is inside the radome. The array's axis and the dipole's orientation are in y direction. The radome base is in $x-y$ plane, and the radome axis is the z axis. The radiation patterns of the array under three configurations are plotted in Fig. 8. The three configurations are: 1) array above plate (without radome); 2) array with plate and radome both presented; and 3) array alone (in free-space). As is expected, the radiations for the three configurations are different significantly. One can also see from the figure that the radiation on the back direction ($\theta = 180^\circ$) is about 30 dB lower compared to the forward direction. This is apparently due to the presence of the conducting plate on the radome base. In addition, due to the interference of the array with the plate and the radome, the side-lobe has increased considerably.

Finally to demonstrate the advantage of using the MLFMA for VIE, numerical experiment is performed using a spherical dielectric shell of various thickness and frequencies to measure the variation of CPU time per iteration and total memory requirement as functions of the number of unknowns. This simulation uses a spherical dielectric shell of radius 1 m. The thickness is chosen as $0.1\lambda_0$, and dielectric constant is $2.56 + 0.5i$. The frequency is varied from 182 MHz to 2.72 GHz (mesh sizes are adjusted to reflect the change in frequency). Fig. 9 shows the plots of CPU time per iteration and memory requirement as functions of the number of unknowns (recorded on HP NCX computer). The plots are in log-log scale. It can be seen from the figure that the slopes of both curves are very close to unit, indicating the $O(N \log N)$ nature of the functions.

IV. SUMMARY

A fast algorithm based on the VIE is described for the analysis of antenna-radome interaction problems. The algorithm is implemented for both tetrahedron and hexahedron meshes, and is applicable to model arbitrarily shaped 3-D radomes. The application of the multilevel fast multipole algorithm significantly

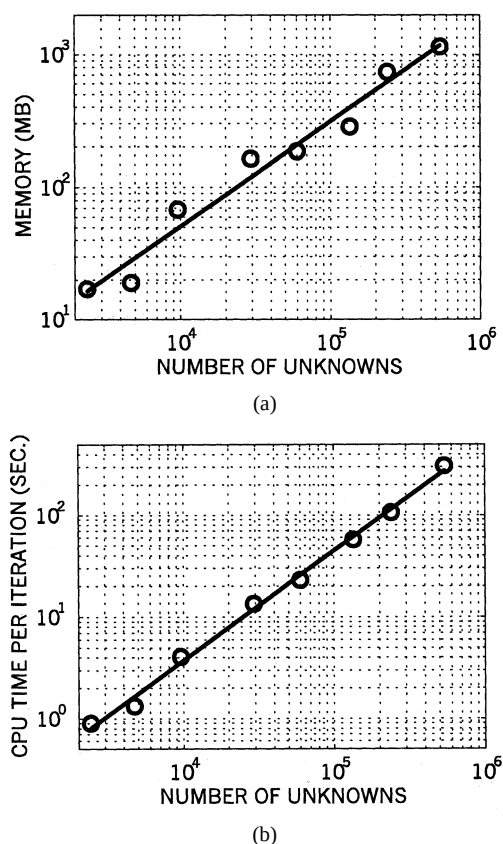


Fig. 9. (a) The memory requirement and (b) CPU time per iteration as functions of the number of unknowns.

reduced the memory and CPU time requirement for solving the VIE, and hence, the algorithm presented in this paper can be applied to problems with electrically large-sized radome. The numerical results agree well with analytical solutions as well as the SIE solutions. This algorithm is different from the ray tracing methods in that it yields highly accurate solution for radomes that contain small-scale geometric variations such as the sharp portion of the ogive radome. The algorithm can be applied to analyze radiation pattern deformation and beam pointing errors when antennas are located inside the dielectric radomes. This algorithm can also be extended to solving the hybrid volume-surface integral equations from which the radome effect on antennas' current distribution and input impedance can be determined.

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