

than 1.1 dB). That is, a CP conical beam is formed. Note that the cross-polarization component (E_L) at $\theta = 34^\circ$ as a function of azimuth angle ϕ is very small, leading to good axial ratios of less than 2.2 dB.

The frequency response for the axial ratio in the direction $(\theta, \phi) = (34^\circ, 0^\circ)$ is shown in Fig. 4, together with the gain. The frequency bandwidth for a 3-dB axial ratio criterion is calculated to be approximately 0.5%. The gain within the same bandwidth is almost constant (approximately 7 dB).

The behavior of the input impedance, $Z_{in} = R_{in} + jX_{in}$, as a function of frequency is shown in Fig. 5. For comparison, the input impedance when the loop is directly fed from a coaxial line (designated as the direct feed), as shown in the inset in Fig. 5, is also presented. It is found that the high-input impedance for the direct feed is reduced to 50 ohms using the EM-coupled feed, by which the antenna is easily matched to a commercially available coaxial line.

III. CONCLUSION

A low-profile loop antenna (antenna height $h = 0.064\lambda_0$), whose circumference is approximately two wavelengths, has been analyzed using the MoM. This loop has a single perturbation element. It is found that the loop radiates a circularly polarized conical beam with a gain of approximately 7 dB over a frequency bandwidth of approximately 0.5%. It is also found that the EM-coupled feed leads to an input impedance of 50 ohms.

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A Coupled Surface-Volume Integral Equation Approach for the Calculation of Electromagnetic Scattering from Composite Metallic and Material Targets

C. C. Lu and W. C. Chew

Abstract—A coupled surface-volume integral equation approach is presented for the calculation of electromagnetic scattering from conducting objects coated with materials. Free-space Green's function is used in the formulation of both integral equations. In the method of moments (MoM) solution to the integral equations, the target is discretized using triangular patches for conducting surfaces and tetrahedral cells for dielectric volume. General roof-top basis functions are used to expand the surface and volume currents, respectively. This approach is applicable to inhomogeneous material coating, and, because of the use of free-space Green's function, it can be easily accelerated using fast solvers such as the multilevel fast multipole algorithm.

I. INTRODUCTION

Recently, advance in fast algorithms for computational electromagnetics demonstrated great potential to solve scattering problems with realistic targets. However, significant reduction of computational complexity has been achieved for scattering problems with conducting targets only. To expand the capability of fast solvers to include material or material coated targets, it is necessary to investigate solvers that can model material-coated targets. This is the motivation for the coupled integral-equation approach presented in this letter. It should be pointed out that scattering calculation for composite conducting and dielectric objects has been studied previously [1]–[4] using coupled surface-integral equations and coupled surface- and volume-integral equations [5]. However, [5] used pulse-basis functions and applies to rectangular shaped objects. In this letter, we present a similar formulation but with general roof-top basis function expansion and the tetrahedron discretization of the dielectrics. Hence, the approach from this letter applicable to arbitrarily shaped objects. In the following formulation, the time factor is $\exp\{-i\omega t\}$ and is suppressed.

II. FORMULATIONS

The problem is to calculate electromagnetic scattering from material-coated objects. The incident wave induces surface current $\bar{J}_S$ on the conducting surface S of the objects, and also induces volume current $\bar{J}_V$ in the dielectric region V . The total scattered field is the superposition of radiations from the surface current and volume current. Let the radiation from $\bar{J}_S$ and $\bar{J}_V$ be $\bar{E}_S^{sca}$ and $\bar{E}_V^{sca}$, respectively, then

$$\bar{E}_\alpha^{sca} = i\omega\mu_b \int_\alpha \bar{\bar{G}}(\bar{r}, \bar{r}') \cdot \bar{J}_\alpha d\bar{r}', \quad \alpha = S \text{ or } V \quad (1)$$

where $\bar{\bar{G}} = (\bar{I} + \nabla\nabla/k_b^2) \exp\{ik_b|\bar{r} - \bar{r}'|\}/4\pi|\bar{r} - \bar{r}'|$ is the 3-D dyadic Green's function, and k_b is the wavenumber for the background media. The surface integral equation is formed based on the boundary

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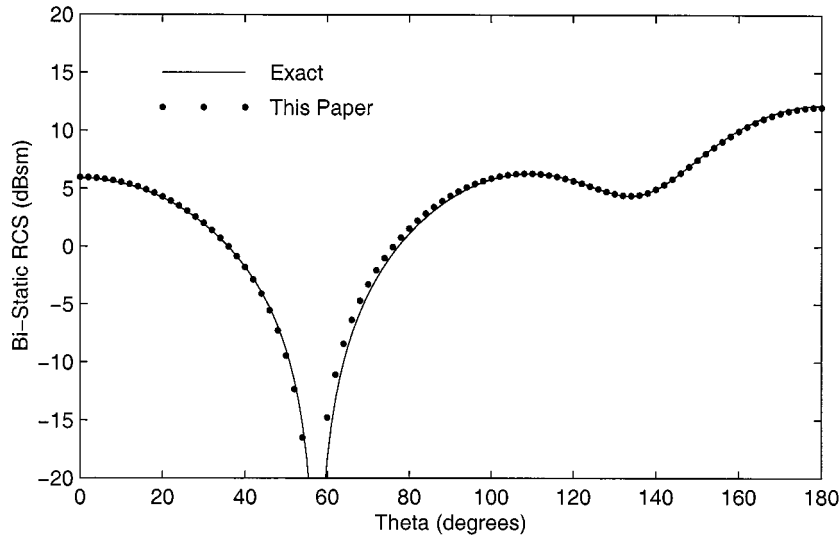


Fig. 1. The calculated RCS of a conducting sphere coated with dielectric material.

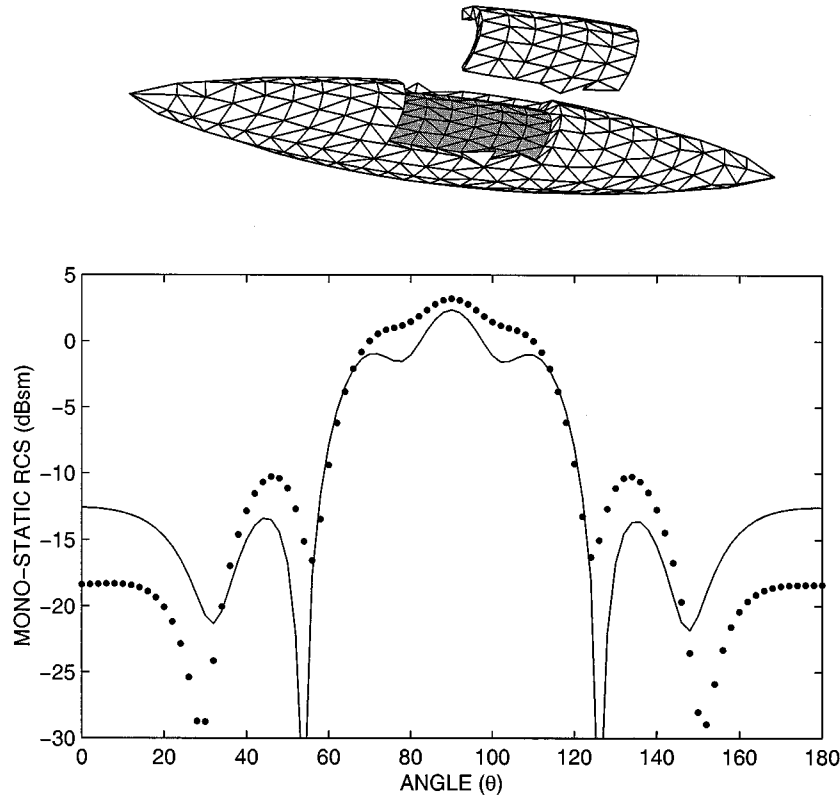


Fig. 2. The calculated RCS of a coated conducting ogive-like object (solid line), and that of a conducting object (dotted line) whose surface is exactly the same as the outermost surface of the coated object. The upper part of the figure shows the geometry mesh.

condition, which requires vanishing tangential component of total electric field

$$\overline{E}_S^{sca}(\vec{r}) + \overline{E}_V^{sca}(\vec{r}) + \overline{E}^i(\vec{r}) = 0, \quad \vec{r} \in S. \quad (2)$$

The volume-integral equation is formed by writing the total electric field $\overline{E}$ in dielectric as the summation of the incident field $\overline{E}^i$ and the scattered field, i.e.,

$$\overline{E}(\vec{r}) = \overline{E}_S^{sca}(\vec{r}) + \overline{E}_V^{sca}(\vec{r}) + \overline{E}^i(\vec{r}). \quad (3)$$

Since the polarization current is related to the total electric field by $\overline{J}_V = i\omega(\epsilon_b - \epsilon)\overline{E}$, we actually have two unknown functions $\overline{J}_S$ and $\overline{J}_V$ in (2) and (3). In the implementation of this letter, $\overline{D} = \epsilon\overline{E}$ is used as the distribution function in the dielectric.

The coupled-integral equations are converted into matrix equations using the method of moments (MoM) [9]. The procedure consists of subdividing the solution domain into small cells, and representing the induced currents by basis functions that are defined on those small cells. In this letter, small triangular patches are used to model the conducting surface, and the tetrahedron is applied to represent the dielectric region. An arbitrarily shaped object can be accurately

modeled when the cell sizes are small enough. In order to simplify the treatment of conductor-dielectric interface, it is required that the triangular patches coincide with the external faces of tetrahedrons. The surface and volume distribution functions are expanded in terms of 3-D vector basis function that was originally introduced by Rao, Wilton, and Glisson (RWG) in [7] and has also been applied for solving volume-integral equations in [10].

The above selection of volume basis function has an advantage that the normal component of $\bar{D}$ is continuous across the faces between cells. This is consistent with the boundary condition for dielectric interfaces. However, since $\bar{D}$ on the exterior boundary is not necessarily zero, additional degrees of freedom must be introduced to determine $\bar{D}$ on the exterior boundary that interfaces with the background media. This is realized by assigning a “half” basis function for each exterior face of the tetrahedral cell. Hence, a “half” basis function is associated with only one tetrahedron. However, if an exterior face of a tetrahedron is terminated by a conducting patch, then this “half” basis can be removed by using the continuity condition. It can be shown that $\hat{n} \cdot \bar{D} = \nabla \cdot \bar{J}_S / i\omega$, and the coefficients associated to those “half” basis functions are related to a set (M_S) of surface basis coefficients by

$$a_n^V = \sum_{m \in M_S} a_m^S \nabla \cdot \bar{f}_m^S(\bar{r}) / (i\omega \hat{n} \cdot \bar{f}_n^V). \quad (4)$$

It should be pointed out that since $\bar{f}_n^V$ is the RWG basis function defined over tetrahedrons, $\hat{n} \cdot \bar{f}_n^V$ is a constant over the tetrahedron surface. Otherwise a testing procedure is needed to determine a_n^V .

III. NUMERICAL RESULTS

In this section two numerical results will be shown. The first result, shown in Fig. 1, is the calculated radar cross section (RCS) of a conducting sphere coated with dielectric material. The radius of the sphere is $0.333\lambda_0$ with $\lambda_0 = 1.5$ m, the coating thickness is $0.0667\lambda_0$, and the dielectric permittivity of the coating material is $\epsilon_r = 4$. The incident angles are $\theta^i = 0$, $\phi^i = 0$, and the observation angles are $\theta^s = 0$ to 180° , and $\phi^s = 0$. The number of triangular cells used to approximate the conducting surface is 432, and the number of tetrahedral cells to model the coating spherical shell is 1296. The exact solution using Mie series is also shown in the figure as reference.

The second example, shown in Fig. 2, is the calculated radar cross section of a coated ogive-like object. The incident plane wave is in V-pol [i.e., $E_\theta^i = 1$ (V/m) and $E_\phi^i = 0$ for the incident wave at the origin]. The conducting part of this object is formed by joining the bases of two cones. The cones are of the same size and shape with base radius $0.24\lambda_0$ at frequency 300 MHz, and height of $1.0\lambda_0$. The cones are formed by rotating a parabola profile around the z axis. Material with $\epsilon_r = 2.56 + i0.5$ is uniformly coated over the conducting surface and the coating thickness is $0.07\lambda_0$. Also, shown in Fig. 2 (dotted line) is the RCS of a conducting object whose surface is exactly the same as that of the outermost surface of the coated object. This comparison is to show the effect of material coating. It can be observed that the RCS for the coated object is smaller in the angle range from $\theta^s = 40^\circ$ to $\theta^s = 140^\circ$. However, in the angle range of $(0, 40^\circ)$ and $(140^\circ, 180^\circ)$ the coated object has larger RCS than the conducting object. In other words, for this specific shape, when the incidence angle is close to the normal direction with respect to the surface, the coating has the effect of reducing the RCS. And if the incidence angle is larger (close to grazing incidence), the coating has an opposite effect, the overall

RCS is increased. This can be attributed to the excitation of surface waves.

IV. SUMMARY

The coupled surface-volume integral equation for electromagnetic scattering from material coated object is solved using the method of moments. The composite metallic and dielectric object is modeled with mixed triangular surface patch and the tetrahedral volume mesh. Accurate result has been obtained for a coated sphere. The approach presented in this paper is suitable for modeling inhomogeneous material or material coatings. It has the same versatility as the finite-element method in modeling material coating. Therefore, it can be an important alternative to the finite-element method. For instance, when the contrast of the dielectric material is low, this method is a perturbation of the free-space metallic scatterer problem, where rapid convergence by iterative solution is expected. Another important feature of this approach is that both the surface integral equation and the volume integral equation are formulated using the free-space Green's function. This feature makes it easy to apply the multilevel fast multipole solvers to reduce the computational complexity.

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