

Electrodynamic Analysis of Combined Microstrip and Coplanar/Slotline Structures with 3-D Components Based on a Surface/Volume Integral-Equation Approach

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Abstract—This paper deals with the electrodynamic analysis of structures embedded in multilayered media, which consist of both microstrip/stripline and coplanar/slotline components also comprising three-dimensional components like vertical interconnects, finite dielectric fillings, or coatings. The analysis is based on a surface/volume integral-equation method using a magnetic surface current description for the slot areas, electric surface currents for the planar microstrip/stripline structures, and electric or polarization volume currents for the description of vertical interconnects and finite dielectric regions. The current discretization is performed by rectangular subdomain basis functions with asymmetric segmentations and corresponding volume current functions on arbitrary nonuniform meshes. For the effective and accurate evaluation of the system matrix, we apply a general asymptotic subtraction technique combined with complete analytical solutions of all dominant asymptotic system matrix entries. Furthermore, we use adaptive database techniques employed to the remaining numerical integrations and identity search algorithms for an optimized redundancy reduction. The method was applied to a large class of different structures comprising customary microstrip components, coplanar bandstop filters, and metal-insulation-metal capacitances and a new class of submillimeter-wave receivers. The results are in good agreement with experimental data or with results of finite-difference approaches, where, however, the latter require much more computational and storage effort.

Index Terms—Convergence acceleration, planar 3-D analysis, redundancy reduction, surface/volume integral equation.

I. INTRODUCTION

IN THE LAST years, a further increase of the complexity and package density of multilayer microwave circuits for applications in mobile communication, navigation, remote sensing systems, etc. can be observed. Besides the monolithic integrated circuits with operating frequencies of some tens of gigahertz, submillimeter-wave applications like new receiver concepts in the radio-astronomy and satellite-based earth observation programs have also recently gained growing interest.

Whereas the microstrip technology still dominates the field of multilayer microwave circuits, a combined employment of

microstrip/stripline and coplanar/slotline components seems to be the best way to meet the specifications of novel concepts, especially at high frequencies. Thus, slot antennas embedded in coplanar or microstrip mixer structures are well suited for frequencies up to the terahertz region due to the weak excitation of surface waves. The realization of monolithic-microwave integrated circuits (MMIC's) in coplanar techniques offers the designer much more flexibility regarding layout and component integration.

For the electrodynamic characterization of such structures, powerful simulation tools are necessary for a reliable and effective design process. However, only a small number of commercial software tools with a full support of coplanar structures is available. From these tools, the HP Momentum program¹ and the computer-aided design (CAD) tool COPLAN [1] have found a larger popularity. The HP Momentum program is based on a surface integral-equation method modeling the slot areas with the duality principle [2], whereas the tool COPLAN is based on a circuit library combined with a quasi-static finite-difference circuit analysis. Since a general full-wave analysis of coplanar components based on local methods like finite-difference time domain (FDTD) or finite-difference frequency domain (FDFD) [3] is very storage and time consuming, integral-equation-based methods seem to be most appropriate and versatile for the characterization of combined structures consisting of both coplanar/slotline and microstrip/stripline circuits and antennas.

Due to the lack of appropriate commercial software packages and the fact that their flexibility is often not sufficient for special problems and expanded simulations, we have developed a concept based on a surface/volume integral-equation system. By using a combined formulation of an electric-field surface integral equation for planar metallizations, a magnetic-field surface integral equation for planes with slot areas and a volume integral equation for vertical metallizations as well as finite dielectric areas, each circuit component can be modeled with a minimum of discretization effort. In contrast to uniform discretization concepts like [4], the structure discretization in our approach is performed by rectangular subdomain basis functions with arbitrary asymmetric segmentations and corresponding volume current basis functions allowing a very

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¹ Momentum, Hewlett-Packard Company (HP Eesof) Westlake Village, CA.

flexible nonuniform meshing. Optionally, different approximations transverse to current flow concerning the planar currents are possible [5]. Another flexible analysis strategies with volume currents, but only for microstrip structures with vertical metallization parts, are given in [6] and [7].

The computation of the system matrix elements is performed in the spectral domain after the analytical evaluation of all space-domain integrations concerning the vertical structure components. Afterwards, the remaining spectral-domain integrations are performed using a general asymptotic subtraction technique splitting the integrals into fast and slow converging parts. For the efficient solution of the slow converging parts, which contain the asymptotic integrand representations, numerous methods have been presented in the literature.

Many contributions, e.g., [4], [8], and especially commercial software tools^{2,3} for two-and-one-half dimensional (2.5-D) analysis, deal with the use of fast Fourier transform (FFT) algorithms. However, the application of such techniques largely requires the utilization of uniform meshing techniques with only simple symmetric rooftop functions, which significantly limits the modeling flexibility. The procedure in [9] is based on numerical integrations in Cartesian coordinates, but the computations remain very time consuming. Other contributions deal with approximate solutions of these integrals in the space domain or with partly analytical solutions [10] based on an integral transform technique, but thus far, only for simple planar microstrip geometries with uniform meshing.

In order to overcome all drawbacks of the methods mentioned above, we apply a technique allowing the determination of complete analytical solutions for the slowly converging integrals forming the asymptotic system matrix.

This technique is based on the partly analytical procedure we have already presented in [11] and [14] in context with planar microstrip structures, but it tends to numerical instabilities in context with large structures. The decisive progress, the development of a procedure to determine complete closed-form solutions for all dominant asymptotic spectral integrals we finally could present in [15], mainly applied to microstrip structures comprising three-dimensional (3-D) components.

Due to the fast convergence of the remaining spectral integrals combined with the exact solution for the asymptotic system matrix, only a small number of sampling points are necessary for the remaining numerical integrations. These circumstances allow the employment of adaptive database algorithms based on the storage and multiple use of Fourier transforms, exponential shift factors, and generalized Green's function components.

Furthermore, we use a redundancy-reduction algorithm for the search of identities within the system matrix. First approaches have also been presented in [11] and [14], and have recently been extended and improved for the combined coplanar/microstrip structure analysis, thus, redundant computations are now completely suppressed.

In what follows, the formulation of the surface/volume integral-equation system is presented, followed by the dis-

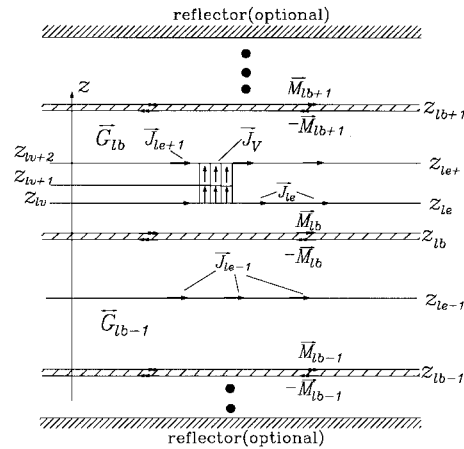


Fig. 1. General structure modeling.

cretization scheme with the basis functions used. Afterwards, the general asymptotic subtraction technique is outlined in some detail. Subsequently, the principles of the database strategies and redundancy reduction are briefly described. The numerical results, which are finally presented, comprise a large class of different structures and field problems starting from 3-D microstrip components over symmetric coplanar/slotline structures to combined microstrip/coplanar circuits.

II. FORMULATION

The considered structures can consist of an arbitrary number of planes containing slot areas within an infinite ground metallization and planes with planar microstrip/stripline components embedded in a multilayered environment. Fig. 1 shows a possible arrangement of the metallization planes and 3-D material distributions modeled by volume current areas $\vec{J}_V$. The layer interfaces d_i , indicating the z -coordinate of the bottom interface of a layer “ i ,” are not explicitly outlined. The microstrip/stripline components in the planes $z = z_{ie}$ are described by electric surface currents $\vec{J}_{ie}$ and the slot areas within the planes $z = z_{ib}$ are modeled by magnetic surface currents $\vec{M}_{ib}$. Furthermore, volume current areas $\vec{J}_V$ can be arbitrarily arranged between the metallization planes. The bottom and top planes of the different volume current areas are designated as z_{lw} , z_{lw+1} , etc. With the use of electric volume currents, vertical interconnects can be described, and additional bounded dielectric inhomogeneities like fillings or coatings are modeled with polarization volume currents.

For solving the corresponding field problem of the structure, on one hand, we have to fulfill the surface impedance boundary condition

$$\vec{E}(x, y, z_{ie})|_{\text{tan}} = Z_{\text{tot}}(x, y, z_{ie})\vec{J}(x, y, z_{ie}) + \vec{E}^{\text{imp}}|_{\text{tan}} \quad (1)$$

on the planes $z = z_{ie}$ carrying microstrip/stripline components, where Z_{tot} is a space-varying surface or line impedance, $\vec{E}^{\text{imp}}$ comprising the excitation by Δ -gap voltage sources or external fields and, on the other hand, we have to fulfill the continuity condition for the magnetic field

$$(\vec{H}^+(x, y, z_{ib}) - \vec{H}^-(x, y, z_{ib}))|_{\text{tan}} = -\vec{e}_z \times \vec{J}_{\text{imp}} \quad (2)$$

² Sonnet Software, Liverpool, NY.

³ Microwave Explorer, Compact Software, Paterson, NJ.

on the ground planes $z = z_{lb}$ with the slot areas. $\vec{J}_{\text{imp}}$ are impressed electric currents, and $\vec{H}^+$ and $\vec{H}^-$ are the magnetic fields above and below the slot plane, respectively. Additionally, we have to fulfill the condition for the electric field within the areas of vertical interconnects and finite dielectric regions

$$\vec{E}(\vec{r}) = \frac{\vec{J}_V(\vec{r})}{j\omega(\epsilon(\vec{r}) - j\kappa(\vec{r})/\omega - \bar{\epsilon})}, \quad \vec{r} \in \text{volume current areas} \quad (3)$$

where $\epsilon(\vec{r})$ and $\kappa(\vec{r})$ describe the space-varying permittivity and conductivity of these areas, $\bar{\epsilon}$ is the permittivity of the surrounding dielectric layer wherein the areas with polarization currents $\vec{J}_V$ are embedded.

With the conditions (1)–(3), we can formulate a system of coupled surface/volume integral equations, which is solved with the Galerkin method of moments.

To get an overview of the integral-equation system, we have to fulfill

$$\begin{aligned} & \left[\vec{H}^+(\vec{M}_{lb}) - \vec{H}^-(\vec{M}_{lb}) - \vec{H}^+(\vec{M}_{lb+1}) - \vec{H}^-(\vec{M}_{lb-1}) \right. \\ & \quad \left. + \vec{H}^+(\vec{J}_{le}) + \vec{H}^+(\vec{J}_{le+1}) + \vec{H}^+(\vec{J}_V) - \vec{H}^-(\vec{J}_{le-1}) \right]_{z=z_{lb}, \tan} \\ & = -\vec{e}_z \times \vec{J}_{\text{imp}, z=z_{lb}} \end{aligned} \quad (4)$$

for the tangential magnetic fields above and below the slot plane z_{lb} with respect to Fig. 1, where $\vec{H}^+(\vec{M}_{lb})$ denotes the magnetic field above the slot plane z_{lb} stemming from the magnetic currents $\vec{M}_{lb}$, etc. (see, e.g., [16]) and

$$\begin{aligned} & \left[\vec{E}(\vec{J}_{le}) + \vec{E}(\vec{J}_{le+1}) + \vec{E}(\vec{J}_V) + \vec{E}(\vec{M}_{lb}) - \vec{E}(\vec{M}_{lb+1}) \right]_{z=z_{le}, \tan} \\ & = Z_{\text{tot}}(x, y, z_{le}) \vec{J}_{le} + \vec{E}_{\text{imp}, z=z_{le}}|_{\tan} \end{aligned} \quad (5)$$

for the tangential electric fields in the plane $z = z_{le}$. We have to formulate similar equations for the fields in the other planes and the volume current areas forming the overall integral-equation system for all unknown electric and magnetic surface currents as well as volume currents.

A. Current Discretization

For the current discretization, we apply the basis functions given in Fig. 2. Fig. 2(a) shows the standard basis function (here, with the current in x -direction) for electric and magnetic surface currents with arbitrary segmentation lengths w_{xl} and w_{xm} . In the initial approaches, we introduced a piecewise sinusoidal approximation in the current direction [5], [11], as it is still shown in the figure, but typically we need a current resolution, where the sinusoidal distribution largely changes into a piecewise linear approximation, thus, the advantage of the sinusoidal description becomes negligible. As outlined later on, this linear approximation is also of decisive importance for the applicability of analytical techniques. Therefore, we now use a piecewise linear description in current flow. Since the space- and spectral-domain descriptions of these basis functions are quite simple, they are omitted here. In Fig. 2(b),

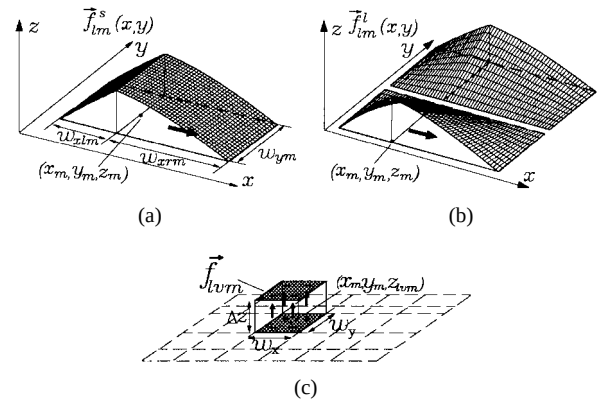


Fig. 2. Applied basis functions.

two further basis functions are additionally shown, which allow a piecewise linear description transverse to the current flow if two basis functions of this kind are composed instead of one function of Fig. 2(a). These functions are especially well suited for the resolution of edge current effects [14] and other high current variations, but their application is combined with a corresponding higher numerical effort.

In Fig. 2(c), the basis function for vertical volume currents, arranged on the mesh of the planar current functions, is outlined. The description in the space domain reads

$$\vec{f}_{lm}(x, y, z) = \text{rect}\left(\frac{x - x_m}{w_{xm}/2}\right) \text{rect}\left(\frac{y - y_m}{w_{ym}/2}\right) \cdot \text{rect}\left(\frac{z - \Delta z/2 - z_{lm}}{\Delta z/2}\right) \vec{e}_z \quad (6)$$

and

$$\begin{aligned} \vec{f}_{lm}(k_x, k_y, z) &= w_{xm} \text{si}\left(k_x \frac{w_{xm}}{2}\right) w_{ym} \text{si}\left(k_y \frac{w_{ym}}{2}\right) \\ &\cdot \text{rect}\left(\frac{z - \Delta z/2 - z_{lm}}{\Delta z/2}\right) e^{jk_x x_m} e^{jk_y y_m} \vec{e}_z \end{aligned} \quad (7)$$

in the spectral domain. The divergence of this basis function shows two associated constant charge cells on the bottom and top faces, which fully corresponds to the divergence within a segment of a planar basis function [see Fig. 2(a)]. Due to this property, the volume current basis functions can represent a generalized polarization current in the vicinity of metallic conductors described by the basis functions [see Fig. 2(a)] with the definition

$$\vec{J}_V(\vec{r}) = j\omega(\epsilon(\vec{r}) - j\kappa(\vec{r})/\omega - \bar{\epsilon}) \vec{E} \quad (8)$$

where $\bar{\epsilon}$ denotes the permittivity of the dielectric layer surrounding the volume current area. $\epsilon(\vec{r})$ and $\kappa(\vec{r})$ represent a possible variation of the permittivity and conductivity within the volume current area, respectively [see also (3)]. If we use thin volume current basis functions (small w_{xm} or w_{ym}) combined with a strongly asymmetric segmentation of the corresponding planar basis functions, we can also apply a quasi-surface current description in the z -direction.

With the introduced basis functions, we get the overall current

$$\vec{J}_{\text{tot}} = \sum_{le}^{L_E} \sum_m^{N_{le}} I_{lem} \frac{\vec{f}_{lem}}{b_{lem}} + \sum_{lb}^{L_B} \sum_m^{N_{le}+N_{lb}} U_{lbm} \frac{\vec{f}_{lbm}}{b_{lbm}} + \sum_{lv}^{L_V} \sum_k^{N_{lv}} I_{lvk} \frac{\vec{f}_{lvk}}{a_{lvk}} \quad (9)$$

where N_{le} is the number of unknown planar electric current amplitudes in the plane le and L_E is the total number of planes with planar electric currents, analogously for planar magnetic currents and volume currents. The coefficients b_{lem} and b_{lbm} correspond to the width of the basis functions [see Fig. 2(a)], a_{lvk} corresponds to the area of the bottom faces of the volume current basis functions. By introducing these coefficients, we get pure current and voltage amplitudes I_{lem} , I_{lvk} , and U_{lbm} within the current representation (9).

B. System Matrix Structure

The discretized integral-equation system is solved with the Galerkin method of moments obtaining the linear system of equations

$$\begin{pmatrix} [Z_{le,le}] & [ZY_{le,lb}] & [Z_{le,lv}] \\ [ZY_{lb,le}] & [Y_{lb,lb}] & [ZY_{lb,lv}] \\ [Z_{lv,le}] & [ZY_{lv,lb}] & [Z_{lv,lv}] \end{pmatrix} \cdot \begin{pmatrix} [I]_{lem} \\ [U]_{lbm} \\ [I]_{lvk} \end{pmatrix} = \begin{pmatrix} [U]_{\text{imp}} \\ [I]_{\text{imp}} \\ 0 \end{pmatrix} \quad (10)$$

where $[Z_{le,le}]$, $[Z_{le,lv}]$, $[Z_{lv,le}]$, and $[Z_{lv,lv}]$ are impedance submatrices of self and mutual couplings between planar electric currents and volume currents, $[Y_{lb,lb}]$ is an admittance submatrix related to self and mutual couplings of magnetic surface currents, and $[ZY_{le,lb}]$, $[ZY_{lb,le}]$, $[ZY_{lb,lv}]$, and $[ZY_{lv,lb}]$ are submatrices related to the mutual couplings of magnetic surface currents with electric and volume currents. Since the elements of these submatrices have no dimension, they are indicated as “ZY.” One of the crucial points of the method of moments lies in the effective and accurate evaluation of the system matrix entries. In order to demonstrate the procedures in our approach, we exemplarily consider the couplings between planar electric currents and volume currents and the couplings of planar magnetic currents under each other. With the definition of the spectral-domain Green’s function, for the electric field (superscript E) from electric currents (index J)

$$\vec{E}(x, y, z) = \int_{z'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{G}_J^E(k_x, k_y, z, z') \cdot \vec{J}(k_x, k_y, z') dk_x dk_y dz' \quad (11)$$

we get for the case of two planar electric basis functions in the planes $z_{le'}$ and z_{le} the matrix entries

$$Z_{nm}(le, le') = \frac{1}{4\pi^2} \int_{k_x} \int_{k_y} \vec{G}_{J,lb}^E(k_x, k_y, z_{le}, z_{le'}) \cdot \frac{\vec{F}_{le'm}(k_x, k_y)}{b_{le'm}} \cdot \frac{\vec{F}_{len}^*(k_x, k_y)}{b_{len}} dk_x dk_y \quad (12)$$

where $G_{J,lb}^E$ denotes the Green’s function, e.g., for regions above the slot plane lb , see also Fig. 1, and analogously for the case of two magnetic basis functions in the same plane z_{lb}

$$Y_{nm}(lb, lb) = \frac{1}{4\pi^2} \int_{k_x} \int_{k_y} \vec{G}_{M,\text{comp}}^H(k_x, k_y, z_{lb}, z_{lb}) \cdot \frac{\vec{F}_{lb,m}(k_x, k_y)}{b_{lb,m}} \cdot \frac{\vec{F}_{lb,n}^*(k_x, k_y)}{b_{lb,n}} dk_x dk_y \quad (13)$$

is obtained, where

$$\vec{G}_{M,\text{comp}}^H(k_x, k_y, z_{lb}, z_{lb}) = \vec{G}_{M,lb}^H(k_x, k_y, z_{lb}, z_{lb}) + \vec{G}_{M,lb-1}^H(k_x, k_y, z_{lb}, z_{lb})$$

is the composite Green’s function of the magnetic-field contributions from above and below the slot plane z_{lb} .

The coupling case of a planar basis function with a volume basis function leads to

$$Z_{nm}(lv, le') = \frac{1}{4\pi^2} \int_{k_x} \int_{k_y} \int_z \vec{G}_{J,lb}^E(k_x, k_y, z, z_{le'}) \cdot \frac{\vec{F}_{le'm}(k_x, k_y)}{b_{le'm}} \cdot \frac{\vec{F}_{lvn}^*(k_x, k_y, z)}{a_{lvn}} dz dk_x dk_y. \quad (14)$$

Finally, we get, in the case of two volume current basis functions, the integral representation

$$Z_{nm}(lv, lv') = \frac{1}{4\pi^2} \int_{k_x} \int_{k_y} \int_z \int_{z'} \vec{G}_{J,lb}^E(k_x, k_y, z, z') \cdot \frac{\vec{F}_{lv'k}(k_x, k_y, z')}{a_{lv'k}} \cdot \frac{\vec{F}_{lvn}^*(k_x, k_y, z)}{a_{lvn}} \cdot dz dz' dk_x dk_y \quad (15)$$

where $\vec{F}_{len}$, $\vec{F}_{lvn}$, and $\vec{F}_{lvn}$ are the Fourier transforms of the n th basis function in the corresponding planes z_{le} , z_{lv} , and z_{lv} , respectively (b_{len} denotes the width of the basis function n in plane le and a_{lvn} denotes the cross-section area of the volume current n on the plane lv). An overview of the components of the different spectral-domain Green’s functions are given in [12], [13], [17], and the Appendix.

The space-domain integrals within (14) and (15), concerning the variable z and z' , can be evaluated analytically. In the case of (14), we get

$$\begin{aligned} \text{IEHV}_{xz}(k_{zi}, z_{le}, z_{lv}) &= \int_{z_{lv}}^{z_{lv}+1} G_{J,xz}^E(k_{zi}, z_{le}, z') dz' \\ &= \frac{jk_x}{2\omega\epsilon_i k_{zi}} \frac{1}{1 - \Gamma_i^{\text{TM}} \hat{\Gamma}_i^{\text{TM}} e^{-j2k_{zi}(d_{i-1}-d_i)}} \\ &\quad \cdot \left\{ \hat{\Gamma}_i^{\text{TM}} [e^{jk_{zi}(z_{lv}-2d_{i-1}+z_{le})} - e^{jk_{zi}(z_{lv}+1-2d_{i-1}+z_{le})}] \right. \\ &\quad + \Gamma_i^{\text{TM}} [e^{jk_{zi}(2d_i-z_{lv}-z_{le})} - e^{jk_{zi}(2d_i-z_{le}-z_{lv}+1)}] \\ &\quad + \Gamma_i^{\text{TM}} \hat{\Gamma}_i^{\text{TM}} [e^{jk_{zi}(2d_i-2d_{i-1}\pm z_{le}\mp z_{lv}+1)} \\ &\quad \left. - e^{jk_{zi}(2d_i-2d_{i-1}\pm z_{le}\mp z_{lv})}] \right. \\ &\quad \left. + e^{\pm jk_{zi}(z_{lv}+1-z_{le})} - e^{\pm jk_{zi}(z_{lv}-z_{le})} \right\} \quad (16) \end{aligned}$$

where “+” is valid for the case $z_{le} \geq z_{lv+1}$ and “-” for the case $z_{le} \leq z_{lv}$. For IEHV_{yz} , only k_x in the first fraction has to be exchanged against k_y . In the case of two volume currents, we have to solve

$$\begin{aligned} \text{IVV}(k_{zi}, z_{lv}, z_{lv'}) \\ = \int_z \left\{ \int_{z_{lv'}}^z G_{J_{zz}}^{E, z \geq z'}(k_{zi}, z, z') dz' \right. \\ \left. + \int_z^{z_{lv'+1}} G_{J_{zz}}^{E, z \leq z'}(k_{zi}, z, z') dz' \right\} dz. \quad (17) \end{aligned}$$

In the most important case in which the volume currents are located on the same corresponding plane $z_{lv} = z_{lv'}$, by carefully consideration of the δ -function in the components $G_{J_{zz}}^E$ (see the Appendix) and utilizing the separation condition $k_x^2 + k_y^2 + k_{zi}^2 = k_i^2$, we finally get

$$\begin{aligned} \text{IVV}(k_{zi}, z_{lv} = z_{lv'}) \\ = \frac{jk_i^2}{\omega\epsilon_i k_{zi}^2} (z_{lv'+1} - z_{lv'}) - \frac{k_x^2 + k_y^2}{k_{zi}^3} \frac{1}{\omega\epsilon_i} \\ \cdot \frac{1}{1 - \Gamma_i^{\text{TM}} \hat{\Gamma}_i^{\text{TM}} e^{-j2k_{zi}(d_{i-1}-d_i)}} \\ \cdot \left\{ \Gamma_i^{\text{TM}} [e^{jk_{zi}(2d_i - z_{lv'+1} - z_{lv'})} \right. \\ \left. - e^{j2k_{zi}(d_i - z_{lv'+1})} - e^{j2k_{zi}(d_i - z_{lv'})}] \right. \\ \left. + \hat{\Gamma}_i^{\text{TM}} [e^{-jk_{zi}(2d_{i-1} - z_{lv'+1} - z_{lv'})} \right. \\ \left. - e^{-j2k_{zi}(d_{i-1} - z_{lv'+1})} - e^{-j2k_{zi}(d_{i-1} - z_{lv'})}] \right. \\ \left. + \Gamma_i^{\text{TM}} \hat{\Gamma}_i^{\text{TM}} [e^{-j2k_{zi}(d_{i-1}-d_i)} \right. \\ \left. - e^{-jk_{zi}(2(d_{i-1}-d_i) + z_{lv'} - z_{lv'+1})}] \right. \\ \left. + 1 - e^{-jk_{zi}(z_{lv'+1} - z_{lv'})} \right\}. \quad (18) \end{aligned}$$

Thus, the elements IEHV_{xz} , IEHV_{yz} , and IVV can be treated as modified representations of the Green's function components $G_{J_{xz}}^E$, $G_{J_{yz}}^E$, and $G_{J_{zz}}^E$ [17]. From similar expressions, derived from the components $G_{J_{xz}}^H$ and $G_{J_{yz}}^H$, we get reactions of volume currents with magnetic currents, denoted as IMHV_{xz} and IMHV_{yz} .

By introducing polar coordinates with $k_\rho = \sqrt{k_x^2 + k_y^2}$, $\varphi = \arctan(k_y/k_x)$, we can represent all matrix elements of (10), shown in (19), at the bottom of this page, where the $\overleftrightarrow{IG}$ denote the generalized Green's functions containing the modified elements IEHV_{xu} , IVV , and IMHV_{xu} . The $\overleftrightarrow{IG}^A$ are asymptotic representations of $\overleftrightarrow{IG}$, which we obtain for $k_\rho \gg k_i$. Subtracting these representations in the first integral of (19)

leads to a fast decaying integrand, thus, the integral becomes well suited for a numerical integration; for the second integral, called the asymptotic part, complete analytical solutions can be determined. For further numerical and analytical treatment, it is important to mention that (19) is modified by four different symmetry relations leading to a reduction of the integration interval to $[0, \pi/2]$ regarding the outer integral [11].

The asymptotic representation for $G_{J_{xx}}^E$ and $G_{J_{xy}}^E$ reads

$$\begin{aligned} G_{J_{uv}}^{E,A} &= G_{J_{uv}}^E|_{k_\rho \gg k_i} \\ &= G_{J_{uv}}^{E, \text{TM}, A}(\varphi) k_\rho + G_{J_{uv}}^{E, \text{TE}, A}(\varphi) \frac{1}{k_\rho} \quad (20) \end{aligned}$$

where $G_{J_{uv}}^{E, \text{TM}, A}(\varphi)$ and $G_{J_{uv}}^{E, \text{TE}, A}(\varphi)$ are given in the Appendix (see, e.g., [9] and [10]). The superscripts TM and TE stem from the subdivision of the Green's functions into TM and TE parts. The asymptotic representations of the composite Green's function in (13) are also given in the Appendix.

The analysis of (16) exhibits the following constant asymptotic representations:

$$\begin{aligned} \text{IEHV}_{xz}^A(z_{le} = z_{lv+1}) &= \frac{\cos \varphi}{2\omega\epsilon_i} (\hat{\Gamma}_i^{\text{TM}, A} - 1) \\ \hat{\Gamma}_i^{\text{TM}, A} &= \frac{\epsilon_{i-1} - \epsilon_i}{\epsilon_{i-1} + \epsilon_i} \quad (21) \end{aligned}$$

$$\begin{aligned} \text{IEHV}_{xz}^A(z_{le} = z_{lv}) &= \frac{\cos \varphi}{2\omega\epsilon_i} (1 - \Gamma_i^{\text{TM}, A}) \\ \Gamma_i^{\text{TM}, A} &= \frac{\epsilon_{i+1} - \epsilon_i}{\epsilon_{i+1} + \epsilon_i}. \quad (22) \end{aligned}$$

The given asymptotic reflection factor $\Gamma_i^{\text{TM}, A}$ must only be considered if the plane z_{lv+1} lies in the interface of two dielectric layers with permittivities ϵ_{i-1} and ϵ_i , correspondingly for the reflection factor $\hat{\Gamma}_i^{\text{TM}, A}$. The analysis of (17) provides the asymptotic representation

$$\text{IVV}^A(z_{lv} = z_{lv'}) = \frac{j}{2k_\rho \omega \epsilon_i} (2 - \Gamma_i^{\text{TM}, A} - \hat{\Gamma}_i^{\text{TM}, A}) \quad (23)$$

with the same considerations for the reflection factors as in the (22). It should be noted that (23) only comprises the dominant asymptotic contribution of (17). The reasons for not considering the complete contribution in this case are outlined later in this paper.

C. Analytical Integration

The analytical integration of the asymptotic parts may be exemplarily demonstrated for planar electric basis functions in a schematic way. Although the first part of the analytical integration was partly shown in [11], a brief repetition of it

$$\begin{aligned} V_{nm} &= \frac{1}{4\pi^2} \underbrace{\int_0^{2\pi} \int_0^{k_{\rho, A}} (\overleftrightarrow{IG}(k_\rho, \varphi) - \overleftrightarrow{IG}^A(k_\rho, \varphi)) \cdot \vec{F}_m(k_\rho, \varphi) \cdot \vec{F}_n^*(k_\rho, \varphi) k_\rho dk_\rho d\varphi}_{\text{numerical integration}} \\ &\quad + \frac{1}{4\pi^2} \underbrace{\int_0^{2\pi} \int_0^\infty \overleftrightarrow{IG}^A(k_\rho, \varphi) \cdot \vec{F}_m(k_\rho, \varphi) \cdot \vec{F}_n^*(k_\rho, \varphi) k_\rho dk_\rho d\varphi}_{\text{complete analytical integration}} \quad (19) \end{aligned}$$

may be essential for the illustration of the complete analytical solution method.

It can be generally shown that all occurring integrals with the asymptotic representations given above have the structure

$$V_{nm}^A(\varphi) = \int_0^\infty \underbrace{\left\{ \sum_\mu A_\mu \frac{\cos(\hat{a}_\mu(\varphi)k_\rho)}{k_\rho^{2i}} + \sum_\nu B_\nu \frac{\sin(\hat{b}_\nu(\varphi)k_\rho)}{k_\rho^{2j-1}} \right\}}_{\text{int}(k_\rho)} \cdot dk_\rho, \quad \text{with } i, j \in [1, \dots, 4] \quad (24)$$

where the coefficients A_μ and B_ν still depend on φ since they contain the corresponding asymptotic representations. $\hat{a}_\mu$ and $\hat{b}_\nu$ depend on basis function parameters as well as on φ . Afterwards, we can write the indefinite integral of $\text{int}(k_\rho)$ in the following form:

$$\begin{aligned} \text{Int}(k_\rho) &= \int \text{int}(k_\rho) dk_\rho \\ &= \underbrace{\sum_p \hat{A}_p \frac{\cos(\hat{a}_p(\varphi)k_\rho)}{k_\rho^{2i-1}} + \sum_q \hat{B}_q \frac{\sin(\hat{b}_q(\varphi)k_\rho)}{k_\rho^{2j-1}}}_{\text{Intsing}(k_\rho)} \\ &\quad + \sum_\mu A_\mu D_{2i} \text{Si}(\hat{a}_\mu(\varphi)k_\rho) \hat{a}_\mu^{2i-1}(\varphi) \\ &\quad + \sum_\nu B_\nu D_{2j-1} \text{Si}(\hat{b}_\nu(\varphi)k_\rho) \hat{b}_\nu^{2j-2}(\varphi), \quad i, j \in [1, \dots, 4] \end{aligned} \quad (25)$$

obtained by multiple integrations by part, where $\text{Intsing}(k_\rho)$ is a very lengthy expression with modified coefficients $\hat{A}_p$, $\hat{B}_q$. D_{2i} and D_{2j-1} are constants without a dimension. We obviously get the definite integral in regard to the k_ρ integration by evaluating the limit value process

$$\int_0^\infty \text{int}(k_\rho) dk_\rho = \lim_{k_\rho \rightarrow \infty} \text{Int}(k_\rho) - \lim_{k_\rho \rightarrow 0} \text{Int}(k_\rho). \quad (26)$$

Due to the singular behavior of the terms in $\text{Intsing}(k_\rho)$ for $k_\rho \rightarrow 0$, a particular analysis of the latter limit value process must be performed. With a multiple use of L'Hospital's rule and due to the existence of all coupling integrals, we can prove that

$$\lim_{k_\rho \rightarrow 0} \text{Intsing}(k_\rho) = 0 \quad (27)$$

holds in any case. Thus, we get very compact analytical solutions for the inner integrals with the schematic structure

$$\begin{aligned} \int_0^\infty \text{int}(k_\rho) dk_\rho &= \lim_{k_\rho \rightarrow \infty} \text{Int}(k_\rho) \\ &= \frac{\pi}{2} \sum_\mu A_\mu D_{2i} \text{sgn}(\hat{a}_\mu(\varphi)) \hat{a}_\mu^{2i-1}(\varphi) \\ &\quad + \frac{\pi}{2} \sum_\nu B_\nu D_{2j-1} \text{sgn}(\hat{b}_\nu(\varphi)) \hat{b}_\nu^{2j-1}(\varphi), \\ &\quad i, j \in [1, \dots, 4]. \end{aligned} \quad (28)$$

The partly analytical solutions of (28) have already been used in [11], but the calculation of (28) leads to numerical instabilities in the case of large structures with fine discretizations.

Therefore, the following steps, leading to complete analytical solutions of all dominant asymptotic parts, are of decisive importance for the efficiency and accuracy of the overall approach.

By sketching the following steps, we may emphasize the coupling of two x -directed basis functions [see Fig. 2(a)]. For this case, after performing the inner integration and an appropriate collection of coefficients and terms, we get the expression

$$\begin{aligned} Z_{nm}^{xx, A} &= \int_0^{\pi/2} Z_{nm}^{xx, A}(\varphi) d\varphi \\ &= \frac{1}{\pi^2} K_{nm}^{xx} \sum_{\mu=1}^9 \frac{\pi}{4} C_{1\mu} \sum_{\nu=1}^4 (-1)^{\nu-1} \\ &\quad \cdot \left(\frac{j}{2\omega\epsilon_i} D_4 \cdot \text{IXX}_{\mu\nu}^{\text{TM}} + \frac{\omega\mu_i}{2j} D_6 \cdot \text{IXX}_{\mu\nu}^{\text{TE}} \right) \end{aligned} \quad (29)$$

where the part integrals $\text{IXX}_{\mu\nu}^{\text{TM}}$ stemming from the TM Green's functions are related to the integral

$$\begin{aligned} \text{IXXP}_{\mu\nu}^{\text{TM}} &= \int \frac{(a_\mu \cos \varphi + b_\nu \sin \varphi)^3}{\cos^2 \varphi \sin^2 \varphi} \\ &\quad \cdot \text{sgn}(a_\mu \cos \varphi + b_\nu \sin \varphi) d\varphi \\ &\quad + \int \frac{(a_\mu \cos \varphi - b_\nu \sin \varphi)^3}{\cos^2 \varphi \sin^2 \varphi} \\ &\quad \cdot \text{sgn}(a_\mu \cos \varphi - b_\nu \sin \varphi) d\varphi. \end{aligned} \quad (30)$$

The coefficients $C_{1\mu}$, K_{nm}^{xx} , D_4 , D_6 , and a_μ and b_ν are given in the Appendix.

We can now derive for the integrands in (30) without the sgn -functions the indefinite integrals

$$\begin{aligned} \text{IXXID}[\pm]_{\mu\nu}^{\text{TM}}(\varphi) &= \int \frac{(a_\mu \cos \varphi \pm b_\nu \sin \varphi)^3}{\cos^2 \varphi \sin^2 \varphi} d\varphi - \frac{a_\mu^3}{\sin \varphi} \\ &\quad \pm 3a_\mu^2 b_\nu \ln(\csc \varphi - \cot \varphi) \\ &\quad + 3a_\mu b_\nu^2 \ln(\sec \varphi + \tan \varphi) \pm \frac{b_\nu^3}{\cos \varphi}. \end{aligned} \quad (31)$$

In context with the determination of the definite integral, we have to consider that one of the sgn functions in (30) changes its sign at $\varphi_{\mu\nu} = \arctan(a_\mu/b_\nu)$. Introducing the abbreviations

$$\text{IXXIDL}_{\mu\nu}^{\text{TM}}(\varphi) = \text{IXXID}[+]_{\mu\nu}^{\text{TM}}(\varphi) + \text{IXXID}[-]_{\mu\nu}^{\text{TM}}(\varphi)$$

and

$$\text{IXXIDU}_{\mu\nu}^{\text{TM}}(\varphi) = \text{IXXID}[+]_{\mu\nu}^{\text{TM}}(\varphi) - \text{IXXID}[-]_{\mu\nu}^{\text{TM}}(\varphi)$$

and considering the subdivision of the integration interval $[0, \pi/2]$ due to the sgn functions, we get the contribution of the definite integral of $\text{IXXP}_{\mu\nu}^{\text{TM}}$ with the help of the relation

$$\begin{aligned} \text{IXX}_{\mu\nu}^{\text{TM}} = & \text{sgn}(a_\mu) \left[\text{IXXIDL}_{\mu\nu}^{\text{TM}}(\varphi_{\mu\nu}) - \lim_{\varphi \rightarrow 0} \text{IXXIDL}_{\mu\nu}^{\text{TM}}(\varphi) \right] \\ & - \text{sgn}(b_\nu) \left[\lim_{\varphi \rightarrow \pi/2} \text{IXXIDU}_{\mu\nu}^{\text{TM}}(\varphi) \right. \\ & \left. - \text{IXXIDU}_{\mu\nu}^{\text{TM}}(\varphi_{\mu\nu}) \right]. \end{aligned} \quad (32)$$

However, it must be noted that the limit values in (32) do not exist if they are considered individually since their integrands have no integrable singularities at $\varphi = 0$ and $\varphi = \pi/2$, which also becomes visible in (31).

However, we can prove that

$$\begin{aligned} \lim_{\varphi \rightarrow 0} \sum_{\mu=1}^9 C_{1\mu} \sum_{\nu=1}^4 (-1)^{\nu-1} \text{IXXIDL}_{\mu\nu}^{\text{TM}}(\varphi) \\ = \lim_{\varphi \rightarrow \pi/2} \sum_{\mu=1}^9 C_{1\mu} \sum_{\nu=1}^4 (-1)^{\nu-1} \text{IXXIDU}_{\mu\nu}^{\text{TM}}(\varphi) = 0 \end{aligned} \quad (33)$$

must hold for the overall limit value processes in any case with a similar line of reasons as for (27). Consequently, the contributions of $\text{IXX}_{\mu\nu}^{\text{TM}}$ to the overall integral read

$$\begin{aligned} \text{IXX}_{\mu\nu}^{\text{TM}} = & \text{sgn}(a_\mu) \left[-\frac{2a_\mu^3}{\sin \varphi_{\mu\nu}} + 6a_\mu b_\nu^2 \ln(\sec \varphi_{\mu\nu} + \tan \varphi_{\mu\nu}) \right] \\ & - \text{sgn}(b_\nu) \left[\frac{2b_\nu^3}{\cos \varphi_{\mu\nu}} + 6a_\mu^2 b_\nu \ln(\csc \varphi_{\mu\nu} - \cot \varphi_{\mu\nu}) \right]. \end{aligned} \quad (34)$$

The solution for the TE parts is based on the integral

$$\text{IXXID}[\pm]_{\mu\nu}^{\text{TE}}(\varphi) = \int \frac{(a_\mu \cos \varphi \pm b_\nu \sin \varphi)^5}{\cos^4 \varphi} d\varphi \quad (35)$$

whose quite extensive analytical solution can be found with computer algebra systems.^{4,5} With an analogous procedure as for the TM parts, we can find

$$\begin{aligned} \text{IXX}_{\mu\nu}^{\text{TE}} = & \text{sgn}(a_\mu) \cdot \text{IXXIDL}_{\mu\nu}^{\text{TE}}(\varphi_{\mu\nu}) \\ & - \text{sgn}(b_\nu) \text{IXXIDU}_{\mu\nu}^{\text{TE}}(\varphi_{\mu\nu}) \end{aligned} \quad (36)$$

with the abbreviations

$$\begin{aligned} \text{IXXIDL}_{\mu\nu}^{\text{TE}}(\varphi) = & 2a_\mu^5 \sin \varphi - 20a_\mu^3 b_\nu^2 \sin \varphi \\ & + 20a_\mu^3 b_\nu^2 \ln(\sec \varphi + \tan \varphi) \\ & + 5a_\mu b_\nu^4 \sin^3 \varphi + 15a_\mu b_\nu^4 \sin \varphi \\ & - 15a_\mu b_\nu^4 \ln(\sec \varphi + \tan \varphi) + 5 \frac{a_\mu b_\nu^4 \sin^5 \varphi}{\cos^2 \varphi} \end{aligned} \quad (37)$$

$$\begin{aligned} \text{IXXIDU}_{\mu\nu}^{\text{TE}}(\varphi) = & -\frac{16}{3} b_\nu^5 \cos \varphi - 10a_\mu^4 b_\nu \cos \varphi \\ & + 40a_\mu^2 b_\nu^3 \cos \varphi + \frac{2}{3} \frac{b_\nu^5 \sin^6 \varphi}{\cos^3 \varphi} \\ & - 2 \frac{b_\nu^5 \sin^6 \varphi}{\cos \varphi} - 2b_\nu^5 \sin^4 \varphi \cos \varphi \\ & - \frac{8}{3} b_\nu^5 \sin^2 \varphi \cos \varphi + 20 \frac{a_\mu^2 b_\nu^3 \sin^4 \varphi}{\cos \varphi} \\ & + 20a_\mu^2 b_\nu^3 \sin^2 \varphi \cos \varphi. \end{aligned} \quad (38)$$

In a similar way, we can derive complete analytical solutions for couplings of basis functions with different current directions and for the other asymptotic parts of the coupling elements in the submatrices $[Z_{lc,lv}]$, $[Z_{lv,lv}]$, and $[Y_{lv,lv}]$. The consideration of asymptotic parts related to couplings of basis functions in different planes is not necessary because the corresponding integrands already show an exponential decay. However, we have also not considered an asymptotic representation related to the first term in (17) and a contribution in context with the coupling of magnetic currents with volume currents, which do not show an exponential decay. We have omitted these terms because we cannot trace back their corresponding asymptotic parts on integrals with the structure of (24); thus, we cannot derive analytical solutions for them. However, on the other hand, numerical investigations have shown that the asymptotic parts of these cases only contribute in a nondominant way to the system matrix. Thus, the omission of the asymptotic subtraction technique for these cases has only a negligible influence on the overall convergence properties of the present approach. In this context it is further remarkable that only the chosen set of basis functions in Fig. 2, but with linear approximation in current flow for the functions shown in Fig. 2(a) and (b), leads to asymptotic parts with the presented complete analytical solutions; if we choose, e.g., a surface current description in the z -direction like in [17], we get dominant asymptotic parts without an analytical solution. Thus, the surface/volume current description is also the optimum choice in this respect.

D. Numerical Integration

On the other hand, it could be shown that, in most cases, the considered dominant asymptotic parts contribute over 90% to the absolute value of a coupling element. Since the accuracy of the asymptotic parts, evaluated by the complete analytical solutions of the previous section, is only restricted by the machine precision, the accuracy of coupling elements is nearly independent of the size of the basis functions and their lateral distances. Together with the fast convergence properties of the first integral in (19), the number of sampling points for the numerical integration can be drastically reduced. The problems with surface and parallel-plate waves, as well as branch points related to the k_ρ -integration, are overcome with the methods already described in [5], [11], and [14]; thus, together with a Gaussian quadrature of maximum order in each interval, only 40–60 sampling points are necessary. Concerning the

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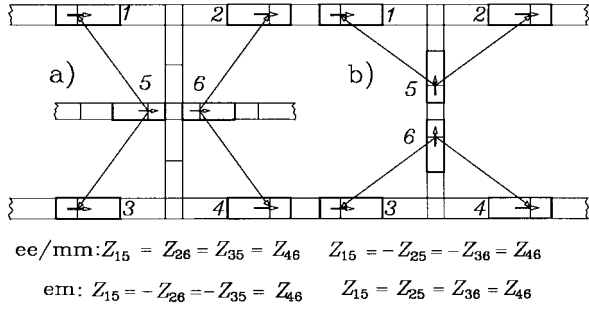


Fig. 3. Examples of identical couplings.

integration with regard to φ , only 12–25 sampling points are needed.

E. Redundancy Reduction Techniques

Due to the displacement invariance of the Green's functions regarding the coordinates x and y and other symmetries, a large number of identical or only phase-shifted matrix elements occur within the system matrix. This is demonstrated in Fig. 3, which presents some examples of matrix identities related to planar basis functions with respect to Fig. 2(a); the relations denoted by “ee/mm” are valid for couplings of both electric current basis functions or both magnetic currents, whereas the relations denoted by “em” are valid for mixed couplings of electric and magnetic current basis functions. If we use a combined employment of all the basis functions in Fig. 2, more than 80 different cases of such identities exist. The identity detection is now performed with the help of a generalized segmentation of the basis functions combined with an appropriate symmetry class subdivision of the different coupling situations and fast text string comparisons. Based on these strategies, a very fast and compact algorithm could be developed that is able to detect any possible matrix identity. Thus, in many cases, only 10%–20% of the matrix elements must be actually computed. In the case of a uniform mesh on a rectangular structure with N basis functions, the maximum redundancy reduction is achieved. In this case, only N different matrix elements of the $N \times N$ system matrix must be computed.

The displacement theorem of the Fourier transformation can be used for a further drastic reduction of the computational effort. Thus, we can write for the first integral in (19)

$$V_{nm1} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{k_{\rho, A}} (\overleftrightarrow{IG}(k_{\rho}, \varphi) - \overleftrightarrow{IG}^A(k_{\rho}, \varphi)) \cdot \vec{F}_{n0}^*(k_{\rho}, \varphi) \cdot \vec{F}_{m0}^*(k_{\rho}, \varphi) e^{jk_{\rho} \cos \varphi \Delta x_{nm}} \cdot e^{jk_{\rho} \sin \varphi \Delta y_{nm}} k_{\rho} dk_{\rho} d\varphi \quad (39)$$

where $\vec{F}_{n,m0}^*(k_{\rho}, \varphi)$ are the Fourier transforms of the non-shifted basis functions, with Δx_{nm} and Δy_{nm} denoting the lateral distances between them. According to the integrand structure, we can first compute and store the different components of $\overleftrightarrow{IG}$ as well as the nonshifted Fourier transforms of the basis functions for later multiple access. Subsequently, all matrix elements that have to be computed are sorted into blocks with same lateral distances Δy_k , thus, we can

compute a vector with all sampling values of the shift factor $e^{jk_{\rho i} \sin \varphi_l \Delta y_k}$ (for a fixed sampling point φ_l). Computing the first block of matrix elements, an array called VDX with the sampling points of all different shift factors $e^{jk_{\rho i} \cos \varphi_l \Delta x_k}$ due to the occurring different lateral distances Δx_k is created. In case of symmetric basis functions (on a piecewise uniform mesh), the shift factors can be reduced to $\sin(k_{\rho} \cos \varphi_l \Delta x_k)$ or $\cos(k_{\rho} \cos \varphi_l \Delta x_k)$.

Since the basis functions are also mainly arranged row-wise in the case of irregular meshes, it is very probable that most of the Δx_k occur again during the computation of the next blocks of matrix entries. In these cases, the precomputed values of the corresponding shift factors can be read out from the array VDX. Based on these principles, we have developed a database algorithm, which allows an optimized system matrix computation in terms of storage and computation effort. For this goal, the matrix creation is simulated at the beginning with assigning each matrix element corresponding addresses for the basis function and shift factor data as well as flags to one of 36 subroutines. In these subroutines, the control of reading, adding, or overwriting of shift factor, Green's function, and Fourier transform data, as well as the optimized integrand collection, is performed within one loop. The reduction of the shift factors in the case of piecewise uniform meshes is also considered. The outlined procedure is then repeated for each sampling point φ_l . To illustrate the possible redundancy reduction, we again bring in the example of the uniform mesh with N basis functions, which demonstrates that only $\approx \sqrt{N}$ different shift factors must be computed. This means that the outlined redundancy reduction strategies are even more effective than FFT-based analysis concepts in this case, together with the further significant difference, not to be restricted on the use of uniform discretization schemes as the latter approaches.

F. Excitation, Input Impedance, and S-Parameter Extraction

For the excitation of the structures, we mainly use δ -gap voltage sources in the case of microstrip-like circuits, and δ -gap current sources in the case of coplanar/slotline structures. If the source area is less than $\lambda/20$, an internal pair of clamps is defined by these sources, and we can directly determine the input impedance at these clamps over the relation U/I . In most cases, the circuit structure is connected with transmission lines, which serve as the input and output ports of the circuit. In these cases, we determine (in a preceding step) the propagation constants and current distribution of the different modes with the help of a one-dimensional eigenvalue/eigenvector analysis of the transmission line. Afterwards, we excite the structure with δ -gap sources or with impressed surface currents (derived from the eigenvector analysis) at the ends of the transmission lines. The incident and reflected waves are extracted with the help of a reference and test plane by using the already known values for the propagation constants from the eigenvalue analysis. Especially in the case of coplanar structures, this method is more reliable than conventional approaches like Prony's method.

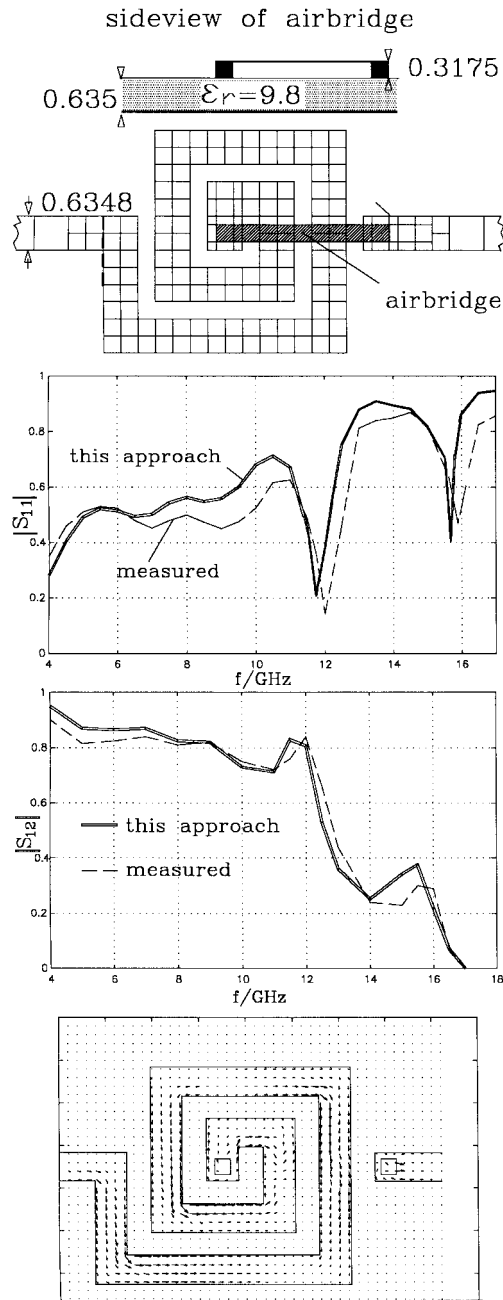


Fig. 4. Analyzed spiral inductor. Top: Geometry and discretization (dimensions in millimeters). Middle: S -parameters. Bottom: Computed current distribution at 16 GHz.

Finally, the S -parameters can be determined by exciting an N -port circuit by N linear independent excitations, leading to a linear system of equations for the unknown S -parameters, similar to those in [4].

III. NUMERICAL RESULTS AND DISCUSSION

Several examples are discussed in this section with the main emphasis being the investigation of coplanar/slotline structures.

A. Spiral Inductor with Airbridge

This example was first analyzed in [4] using a uniform discretization with high resolution. In [7], a triangular meshing

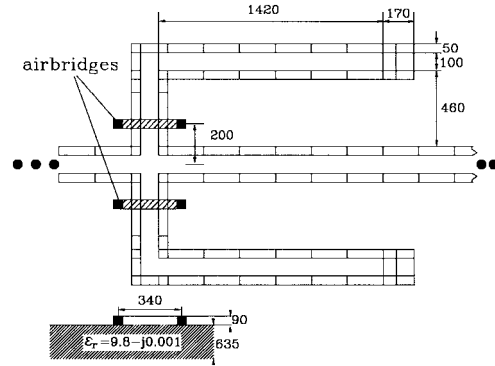


Fig. 5. Structure and discretization of a coplanar bandstop filter (dimensions in micrometers).

was used. In order to test the robustness of our approach, we have chosen a quite coarse discretization. The feeding lines were modeled without subdividing the width, the spiral conductors are subdivided uniformly with a nonuniform meshing around the bond wires of the air bridge. Each bond wire was only modeled with one volume current element, leading to 279 basis functions altogether [Fig. 4, top]. Despite the coarse discretization, we already achieve a good agreement with measured scattering parameters, as shown in the middle of Fig. 4 with only 16-s computation time for the system matrix creation per frequency. The current distribution for 10 GHz is given on the bottom. The calculations in [4] and [7], performed with up to 1300 unknowns, show only a slightly better agreement with respect to the locations of the parasitic resonances at 12 and 16 GHz.

B. Hybrid Coplanar Bandstop Filter

As a second example, we present the simulation of a hybrid bandstop filter for a 18/36-GHz frequency doubler, first investigated by an FDTD approach in [18].

Fig. 5 shows the configuration with the used slot discretization. The first simulation was performed without the air bridges over the bend stubs; as shown in Fig. 6, we cannot recognize any bandstop behavior in this case since the bend stubs act as meander slotlines without the bridges.

Introducing the air bridges, we can now observe the desired bandstop behavior at ≈ 18 GHz combined with a small reflection at 36 GHz. The resonance phenomenon at ≈ 26 GHz is due to the still noncompensated bends of the stubs. Further investigations have shown that this effect can be suppressed by further air bridges or by a modified filter design in monolithic coplanar technology. The agreement of our approach with the measurements is similar or slightly better than the results of the FDTD method; additionally, the FDTD method needs significantly more computation time and resources than our approach, with about 15 s per frequency point.

C. Coplanar Metal–Insulator–Metal Capacitances

The following example shows the behavior of coplanar metal–insulator–metal (MIM) capacitances. Fig. 7, top, shows the configuration of the structure, consisting of a center conductor of width w_c deposited with a thin dielectric film with

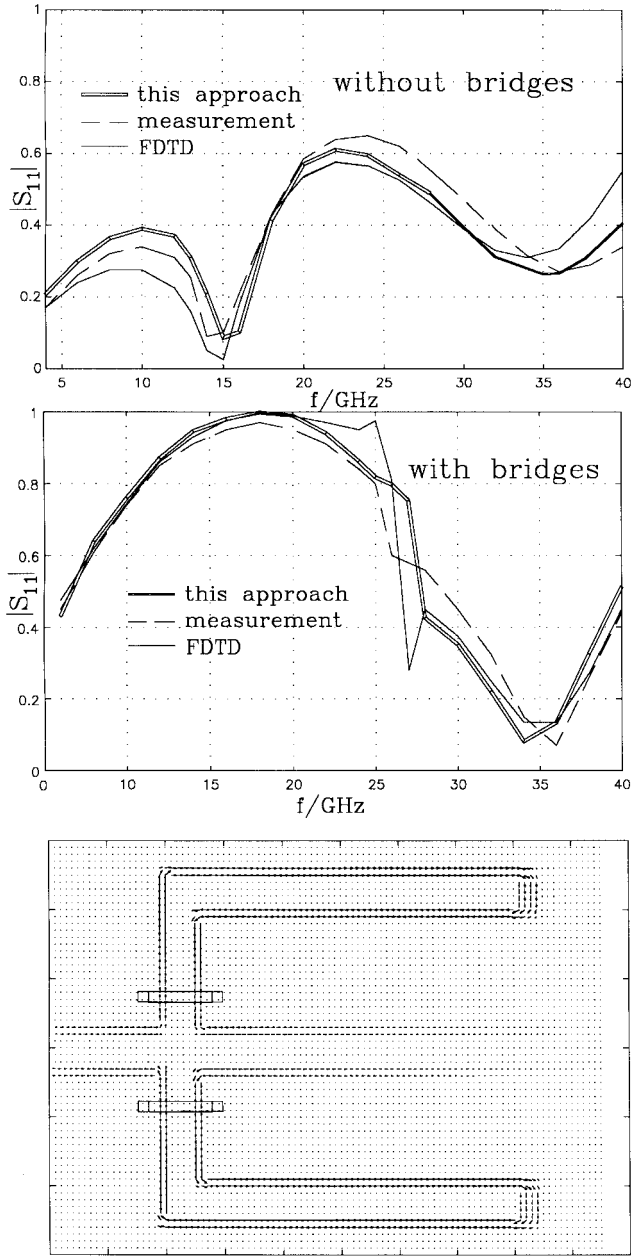


Fig. 6. Reflection factors of the coplanar bandstop filter with and without air bridges and magnetic current distribution at 10 GHz (with air bridges).

$\epsilon_r \approx 6$ and thickness $t_d \approx 0.2 \mu\text{m}$ ($s = 15 \mu\text{m}$, $w = 20 \mu\text{m}$), modeled with polarization volume currents. On top of the dielectric film, a second metallization plane is mounted and connected with the center conductor by an airbridge ($l_g = 15 \mu\text{m}$). Fig. 7, bottom, shows the results for two examples of w_c , where $w_c \cdot l_c = \text{const}$. Although we get some convergence problems due to the very low air bridge (influence of image sources caused by the ground plane), only a few seconds of computation time is needed per frequency point. The results are compared with FDFD computations from [19]. The small differences may be due to material and radiation losses, which are not considered by the FDFD analysis.

The FDFD analysis takes significantly more calculation time (up to 1 h) for this example than our approach with less than 10 s per frequency point.

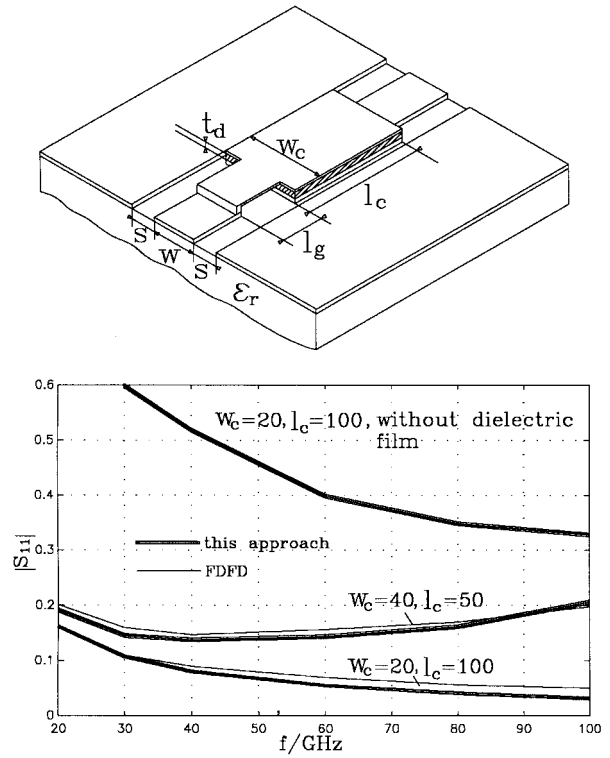


Fig. 7. Structure (top) and simulation (bottom) of coplanar MIM capacitors (dimensions in micrometers).

D. Integrated Submillimeter-Wave Receivers

The analysis and design of satellite-based integrated submillimeter-wave receivers in context with an international collaboration under the management of the European Space Agency (ESA) was one of the main application fields of our approach in recent time. The investigated receivers consist of a hemispherical silicon lens, which forms with the following GaAs wafer an extended hemispherical lens geometry. In the middle of the GaAs wafer is mounted a slot antenna/coplanar mixer arrangement.

After several modifications, the final arrangement in Fig. 8, top, has been chosen, consisting of a ringslot antenna in the middle, a coplanar IF filter on the left-hand side, and a so-called quasi-vertical mixer diode, connected with a short coplanar feeding line to the right-hand side. The main problem concerning the preceding concepts of such coplanar/slotline open-structure receivers was given by parasitic radiation effects caused by the mixer and filter components.

As the computed magnetic current distribution in the middle of Fig. 8 shows, the critical currents at the QVD diode and at the first step of the IF filter are sufficiently low concerning the present structure, resulting in a satisfactory radiation pattern in combination with a synthesized elliptical lens (Fig. 8, bottom). These patterns are computed by a subsequent raytracing/aperture integration method supporting arbitrary extended hemispherical and elliptic lens geometries [20], [21]. Gaussian beam efficiency calculations have also been performed with efficiencies of up to 90% for the overall structure. The structure shows good matching capabilities as well. Thus, an adjustment of the length L_{IF} in combination

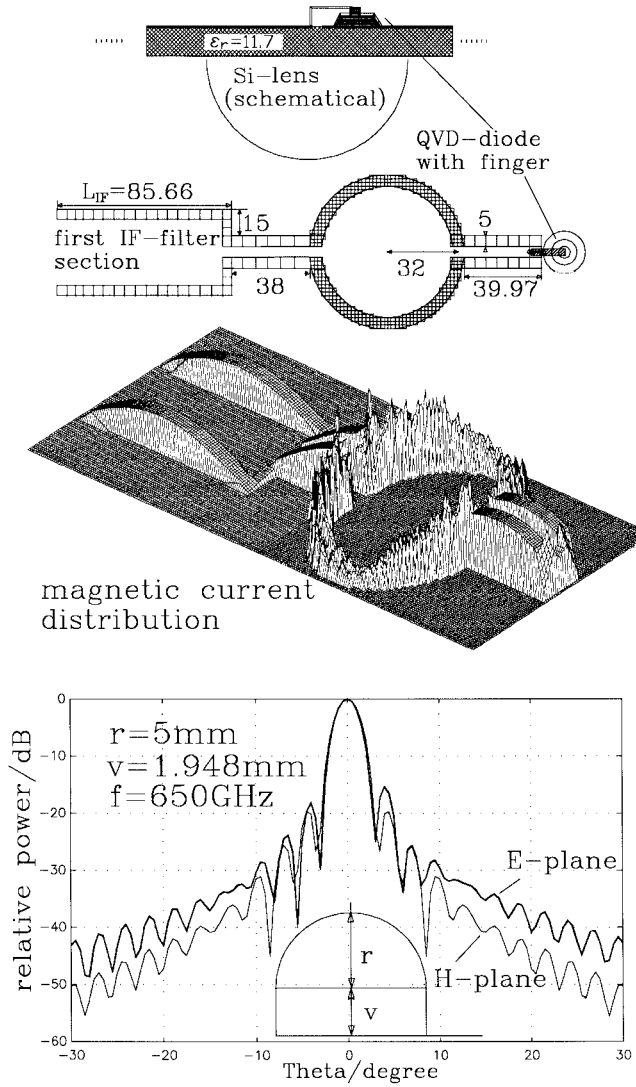


Fig. 8. Component arrangement, discretization, magnetic current distribution, and radiation pattern of an integrated submillimeter-wave receiver with ringslot antenna and coplanar mixer/IF filter (dimensions are in micrometers).

with a preceding adjustment of the feeding line length allows a nearly perfect matching of the diode impedance (about $(25 + j20)\Omega$) concerning the theoretical model. Measurements with scaled models at 65 GHz have meanwhile confirmed the good overall properties of the concept [22].

IV. CONCLUSIONS

The presented electrodynamic simulation concept, which is based on a surface/volume integral equation approach, allows the modeling and analysis of a large class of circuit structures embedded in a multilayered medium. The chosen set of basis functions for the description of planar electric and magnetic currents, as well as for 3-D volume current areas, allows, on one hand, an optimized discretization of each circuit component and, on the other hand, is absolutely necessary for the application of a generalized asymptotic subtraction technique based on complete analytical solutions for the dominant asymptotic system matrix. The overall computational effort and the effort for the remaining numerical integrations

could be drastically reduced by a new identity determination algorithm and an adaptive database strategy. The method has been applied to microstrip components and particularly to general coplanar structures and new integrated submillimeter-wave receiver concepts.

APPENDIX

The asymptotic representations of (20) read

$$G_{Jxx}^{E, TM, A}(\varphi) = \frac{j \cos^2 \varphi}{2\omega\epsilon_i} (1 - \Gamma_i^{TM, A})$$

$$G_{Jxx}^{E, TE, A}(\varphi) = \frac{\omega\mu_i \sin^2 \varphi}{r2j} \quad (40)$$

$$G_{Jyy}^{E, TM, A}(\varphi) = \frac{j \sin^2 \varphi}{2\omega\epsilon_i} (1 - \Gamma_i^{TM, A})$$

$$G_{Jyy}^{E, TE, A}(\varphi) = \frac{\omega\mu_i \cos^2 \varphi}{2j} \quad (41)$$

$$G_{Jxy}^{E, TM, A}(\varphi) = \frac{j \cos \varphi \sin \varphi}{2\omega\epsilon_i} (1 - \Gamma_i^{TM, A})$$

$$G_{Jxy}^{E, TE, A}(\varphi) = \frac{\omega\mu_i \cos \varphi \sin \varphi}{2j} \quad (42)$$

where the asymptotic reflection factor $\Gamma_i^{TM, A} = (\epsilon_{i+1} - \epsilon_i)/(\epsilon_{i+1} + \epsilon_i)$ must only be considered if the basis functions are located in the interface of two layers with permittivities ϵ_{i+1} and ϵ_i . For the composite Green's function $\vec{G}_{M, comp}^H(k_x, k_y, z_b, z_b)$ in (13), we get

$$G_{Muv}^{H, A} = G_{Muv}^H |k_\rho \gg k_i$$

$$= G_{Muv}^{H, TM, A}(\varphi) \frac{1}{k_\rho} + G_{Juv}^{E, TE, A}(\varphi) k_\rho \quad (43)$$

with

$$G_{Mxx}^{H, TM, A}(\varphi) = -j\omega(\epsilon_p + \epsilon_m) \sin^2 \varphi$$

$$G_{Mxx}^{H, TE, A}(\varphi) = \frac{2j \cos^2 \varphi}{\mu_i \omega} \quad (44)$$

$$G_{Myy}^{H, TM, A}(\varphi) = -j\omega(\epsilon_p + \epsilon_m) \cos^2 \varphi$$

$$G_{Myy}^{H, TE, A}(\varphi) = \frac{2j \sin^2 \varphi}{\mu_i \omega} \quad (45)$$

$$G_{Mxy}^{H, TM, A}(\varphi) = j\omega(\epsilon_p + \epsilon_m) \cos \varphi \sin \varphi \quad (46)$$

$$G_{Mxy}^{H, TE, A}(\varphi) = \frac{2j \cos \varphi \sin \varphi}{\mu_i \omega} \quad (47)$$

where ϵ_p and ϵ_m denotes the permittivities directly above and below the slot plane z_b , respectively.

The Green's function component G_{Jzz}^E in (17) ($z > z'$) reads

$$G_{Jzz}^{E, TM} = \frac{-1}{2\omega\epsilon_i} \frac{k_x^2 + k_y^2}{k_{zi}} e^{-j(k_x x' + k_y y')} \cdot \frac{1 + \Gamma_i^{TM} e^{j2k_{zi}(d_i - z')}}{1 - \Gamma_i^{TM} \hat{\Gamma}_i^{TM} e^{-j2k_{zi}(d_{i-1} - d_i)}} \cdot \hat{\Gamma}_{i,j}^{TM} [\hat{\Gamma}_j^{TM} e^{-jk_{zj}(2d_{j-1} - z - z')} + e^{-jk_{zj}(z - z')}] - \frac{1}{j\omega\epsilon_i} e^{-j(k_x x' + k_y y')} \delta(z - z') \quad (48)$$

μ, ν	$C_{1\mu}$	a_μ	b_ν
1	$-w_{xm} \cdot w_{xrn}$	$-(w_{xln} + \Delta x)$	$\left(\frac{w_{yn} - w_{ym}}{2} + \Delta y \right)$
2	$-w_{xlm} \cdot w_{xn}$	$-(w_{xrm} + \Delta x)$	$\left(-\frac{w_{ym} + w_{yn}}{2} + \Delta y \right)$
3	$-w_{xm} \cdot w_{xln}$	$(w_{xrm} + \Delta x)$	$\left(\frac{w_{ym} - w_{yn}}{2} - \Delta y \right)$
4	$-w_{xrm} \cdot w_{xn}$	$(w_{xlm} + \Delta x)$	$\left(\frac{w_{ym} + w_{yn}}{2} + \Delta y \right)$
5	$w_{xrm} \cdot w_{xrn}$	$(w_{xlm} - w_{xln} + \Delta x)$	
6	$w_{xlm} \cdot w_{xln}$	$(w_{xrn} - w_{xrm} + \Delta x)$	
7	$w_{xlm} \cdot w_{xrn}$	$(-w_{xrm} - w_{xln} + \Delta x)$	
8	$w_{xrm} \cdot w_{xln}$	$(w_{xlm} + w_{xrn} + \Delta x)$	
9	$w_{xm} \cdot w_{xn}$	Δx	

with i, j being source and observation layer, respectively, and $d_{i,j}$ being the coordinates of the lower layer interfaces of the corresponding layers. Additionally, G_{Jxz} in (16) reads for $z \geq z'$

$$G_{Jxz}^{E, TM} = \frac{1}{2\omega\epsilon_i} \frac{k_x k_{zj}}{k_{zi}} e^{-j(k_x x' + k_y y')} \cdot \frac{1 + \Gamma_i^{TM} e^{j2k_{zi}(d_i - z')}}{1 - \Gamma_i^{TM} \hat{\Gamma}_i^{TM} e^{-j2k_{zi}(d_{i-1} - d_i)}} \cdot \hat{T}_{i,j}^{TM} \left[\hat{\Gamma}_j^{TM} e^{-jk_{zj}(2d_{j-1} - z - z')} - e^{-jk_{zj}(z - z')} \right] \quad (49)$$

where the TE parts of the above components are zero. Further details and an overview of other components are given in [13] and [17].

The coefficients in (29) are shown at the top of this page, with $\Delta x = x_n - x_m$, $\Delta y = y_n - y_m$, and $K_{nm}^{xx} = 1/(w_{xlm}w_{xrm}w_{xln}w_{xrn})$, $D_4 = 1/6$, and $D_6 = -1/120$.

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