

A New Vector Element in the Volume Integral Equation Method for Nonlinear Magnetostatics

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Abstract -- To calculate nonlinear magnetostatics a new vector element in the volume integral equation has been developed. The magnetization vector in an element is expressed by using co- and contra-variant vectors corresponding to the each surface of the hexahedron element. To accelerate the conversion process, the iteration procedure has been divided into initialization and convergence parts. The method has been applied to TEAM workshop problem 13 of IEEE. Its calculated results agree with the measured data within 2%, indicating the method provides good accuracy.

I. INTRODUCTION

To calculate magnetostatics in magnetic substances, many calculation methods have been proposed such as finite element, boundary element and volume integral methods[1-5]. A volume integral method for magnetostatics has been developed by Newman et al.[1]. It has the following advantages: far-field boundaries are treated automatically; computed fields satisfy Maxwell's equations exactly; and only magnetic substance regions have to be discretized. The location of evaluation points for magnetic intensity in the paper has been selected at the center of the element and the number of the evaluation points is one in each element. This method gives a large calculation error, because the magnetization in an element becomes constant and cannot be expressed the large change of magnetization in the element. Additionally, increase of the number of evaluation points causes reduction of numerical error, such as linear elements or quadratic elements. However, these elements require much memory and CPU time. To overcome these difficulties, Nojima and Ohba[3] proposed a modified constant interpolation method which has six unknowns in an element with respect to each surface of a hexahedral element. Kettunen and Turner[4] proposed the combination method of an edge element and a tree-cotree structure. Nojima and Ohba give a method using a facet element for a hexahedron while Kettunen and Turner have adopted the tetrahedron element and H-formulation. These methods can reduce the number of unknowns and CPU time. But, their calculation results strongly depend on the direction of vectors for unknowns. This gives poor results when the element shape is strongly deformed. We have overcome these problems by adopting co- and contra-variant vectors for a magnetic moment parallel to the principal hexahedron axis.

II. BASIC EQUATIONS

A magnetic field is expressed by the following Maxwell equations:

$$\text{div } \mathbf{B} = 0 \quad (1)$$

$$\text{rot } \mathbf{H} = \mathbf{i} \quad (2)$$

where $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{i}$ are magnetic flux density, magnetic intensity, and current density, respectively. Using permeability for space μ_0 and permeable substances μ , we give the constitutive equation between magnetic flux density and magnetic intensity as follows:

$$\mathbf{B} = \mu_0 \mu \mathbf{H}. \quad (3)$$

The relationship between magnetic flux density and magnetization $\mathbf{M}$ is expressed by

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}). \quad (4)$$

The magnetic intensity $\mathbf{H}$ is decomposed into magnetic intensity by magnetic substances $\mathbf{H}_m$ and by coils $\mathbf{H}_c$,

$$\mathbf{H} = \mathbf{H}_m + \mathbf{H}_c \quad (5)$$

The integral expression of magnetic intensity by magnetic substances is as follows:

$$\mathbf{H}_m = \int \mathbf{M} \cdot \mathbf{n} \text{ grad } \psi d\Gamma - \int \text{div } \mathbf{M} \text{ grad } \psi d\Omega \quad (6)$$

where $\mathbf{n}$ and ψ are the normal vector of the magnetic substance surface and Green's function, respectively. Magnetic intensity due to the exciting coils can be calculated by Biot-Savart's law. Substituting (6) into (5), (4) and (3) yields (7).

$$\mathbf{M} + (1 - \mu) \left(\int \mathbf{M} \cdot \mathbf{n} \text{ grad } \psi d\Gamma - \int \text{div } \mathbf{M} \text{ grad } \psi d\Omega \right) = (\mu - 1) \mathbf{H}_c \quad (7)$$

Magnetization distribution in the magnetic substances is calculated by solving the above integral equation.

III. DISCRETIZATION

To solve (7), magnetic substances are divided into many elements. The hexahedron element is used for the calculation. We have selected the linear interpolation function and six evaluation points for the magnetization vector in an element. The direction of the covariant vector is defined by the following equation corresponding to the element surface:

$$\mathbf{e}_i = \begin{cases} a_i \frac{\partial \mathbf{r}}{\partial \xi} & \cdots & i = 1, 2 \\ a_i \frac{\partial \mathbf{r}}{\partial \eta} & \cdots & i = 3, 5 \\ a_i \frac{\partial \mathbf{r}}{\partial \zeta} & \cdots & i = 4, 6 \end{cases} \quad (8)$$

where ξ , η , ζ indicate nondimensional coordinates in the volume element as illustrated in Fig.1 and a_i is a normalization factor. The contravariant vector corresponding to (8) becomes

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$$\mathbf{e}^i = \begin{cases} \frac{1}{a_i} \nabla \zeta & \dots \quad i = 1, 2 \\ \frac{1}{a_i} \nabla \eta & \dots \quad i = 3, 5 \\ \frac{1}{a_i} \nabla \xi & \dots \quad i = 4, 6 \end{cases} \quad (9)$$

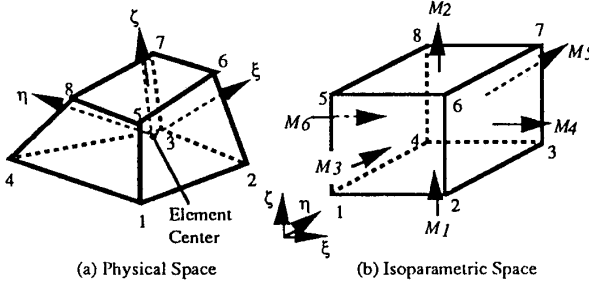


Fig.1 Hexahedron Isoparametric Element

The magnetization vector in an element can be expressed by using the shape function N_j^* :

$$\mathbf{M} = \sum_{j=1}^6 M_j N_j^* \mathbf{e}^j \quad (10)$$

$$N_j^* = \frac{1}{2} (1 + \xi_j \xi) (1 + \eta_j \eta) (1 + \zeta_j \zeta) \quad (11)$$

where ξ_j , η_j and ζ_j are defined as follows.

$$(\xi_j, \eta_j, \zeta_j) = \begin{cases} (0, 0, -1) \dots j = 1 \\ (0, 0, 1) \dots j = 2 \\ (0, -1, 0) \dots j = 3 \\ (1, 0, 0) \dots j = 4 \\ (0, 1, 0) \dots j = 5 \\ (-1, 0, 0) \dots j = 6 \end{cases} \quad (12)$$

Substituting (10) into (7) and multiplying covariant vector $\mathbf{e}_i$ we get the following equation:

$$\sum_{j=1}^6 M_j N_j^* \mathbf{e}^j \cdot \mathbf{e}_i + (1 - \mu) \sum_m \left\{ \sum_{k=1}^6 \int_{\Delta \Gamma_k} \sum_{j=1}^6 M_j N_j^* \mathbf{e}^j \cdot \mathbf{n}_k \mathbf{e}_i \cdot \text{grad} \psi d\Gamma - \int_{\Delta \Omega_m} \text{div} \mathbf{M} \mathbf{e}_i \cdot \text{grad} \psi d\Omega \right\} = (\mu - 1) \mathbf{e}_i \cdot \mathbf{H}_c \quad (13)$$

where m , k , $\Delta \Gamma$ and $\Delta \Omega$ indicate element number, surface number in an element, and surface and volume integrations, respectively. To perform the volume integral term in (13), it has been deformed as follows. Since, magnetization $\mathbf{M}$ in an element is interpolated by linear function, its divergence becomes constant. Therefore, by using an average magnetic charge σ_k on the element surface, the following equation is obtained:

$$\text{div} \mathbf{M} = C = \frac{1}{\Delta V_m} \sum_{k=1}^6 \sigma_k \Delta S_k \quad (14)$$

where ΔV_m and ΔS_k indicate the volume of element m and area of k -th element surface, respectively. On the other hand, the volume integral term of (13) can be transformed to a surface integral because of the constant $\text{div} \mathbf{M}$.

$$\int_{\Delta \Omega_m} \text{div} \mathbf{M} \mathbf{e}_i \cdot \text{grad} \psi d\Omega = \sum_{l=1}^6 \int_{\Delta \Gamma_l} \text{div} \mathbf{M} \mathbf{e}_i \cdot \mathbf{n}_l \psi d\Gamma \quad (15)$$

The average magnetic charge can be assumed to be expressed by magnetization at the center of the element surface.

$$\sigma_k = \sum_{j=1}^6 M_j N_j^* \mathbf{e}^j \cdot \mathbf{n}_k \quad (16)$$

Substituting (14) and (16) into (15), we obtain the following equation.

$$\int_{\Delta \Omega_m} \text{div} \mathbf{M} \mathbf{e}_i \cdot \text{grad} \psi d\Omega = \frac{1}{\Delta V_m} \sum_{l=1}^6 \int_{\Delta \Gamma_l} \sum_{k=1}^6 \sum_{j=1}^6 M_j N_j^* \mathbf{e}^j \cdot \mathbf{n}_k \Delta S_k \mathbf{e}_i \cdot \mathbf{n}_l \psi d\Gamma \quad (17)$$

The final discretization form, using the matrix expression is:

$$[E_{ij} + (1 - \mu_i) K_{ij}] \{M_j\} = (\mu_i - 1) \{H_{ci}\} \quad (18)$$

IV. NONLINEAR CALCULATION METHOD

To calculate nonlinear magnetic substances, the direct iteration process has been modified. This process is constructed of two parts, one is the initialization process and the other is the conversion process.

1) *Initialization process*: If the $n-1$ step permeability μ^{n-1} is given, the next step permeability is estimated as illustrated in Fig.2. (a) The $n-1$ step magnetic intensity H^{n-1} and magnetization M^{n-1} are calculated by Eq.(18). (b) The magnetization M^* is calculated by H^{n-1} and M-H curve. (c) Magnetic intensity H^{**} at the cross point of the lines $M = (\mu^{n-1} - 1)H$ and $H = K_{ii}(M - M^{n-1}) + H^{n-1}$ is calculated. (d) After that the next step permeability μ^n is calculated, where K_{ii} is the diagonal term of matrix K_{ij} .

This approach is equivalent to replacing the solved large matrix into the calculation of the cross point by using only

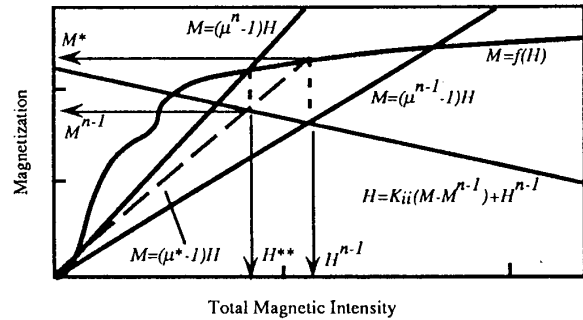


Fig.2 Magnetization Curve and Iteration Process

the diagonal terms of matrix. Then the iteration process can be accelerated.

2) *Convergence process*: If the magnetization curve has strong nonlinearity, the above process cannot get the calculation to converge. Therefore, the following iteration process is added:

$$M^* = f(H^{n-1})$$

$$\mu^* = M^* / H^{n-1} + 1$$

$$\mu^n = \begin{cases} \mu^* & \dots \text{if } (\mu^* - \mu^{n-1})(\mu^{**} - \mu^{n-2}) \geq 0 \\ 0.5(\mu^* + \mu^{n-1}) & \dots \text{if } (\mu^* - \mu^{n-1})(\mu^{**} - \mu^{n-2}) < 0 \end{cases} \quad (19)$$

where μ^{**} indicates permeability calculated by M^* at the $n-1$ step.

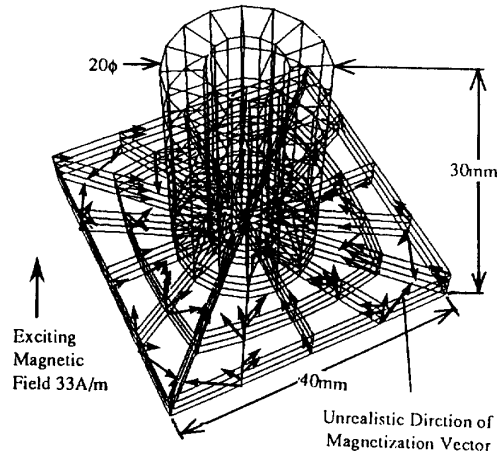
V. CALCULATION RESULTS

The method has been applied to two magnetic problems, the model illustrated in Fig.3 (Case 1) and the TEAM Workshop Problem 13 (Case 2).

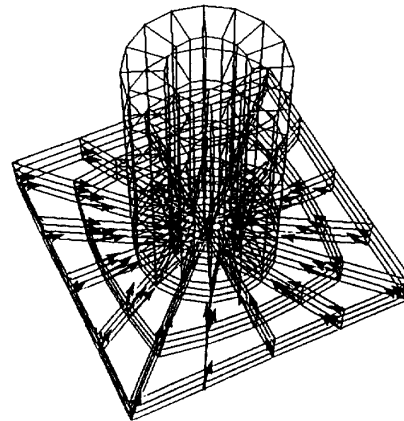
Case 1: Calculation conditions for Case 1 were as follows. Exciting magnetic intensity was 33A/m and permeability was 1000. Calculation results are shown in Figs. 3 (a) and (b). Figure 3 (a) indicates the results for a conventional basis of the magnetization vector. Directions of the calculated vectors are unrealistic because of mesh shape deformation. On the other hand in Fig.3 (b), the present method gives realistic results in spite of a distorted mesh.

Case 2: Calculation conditions are shown in Fig.4. The magnetization curve of the steel plate is illustrated in Fig.5. The magnetic intensity was expressed in a logarithmic scale. As shown in Fig.5 the shape of the M-H curve has a double S character, which makes the convergence for a nonlinear calculation poor. As shown in Fig.4, element division near both ends of the center plate is finer than other parts of the plate. The numbers of elements and unknowns are 1544 and 9264, respectively. The modified iteration process was used for the nonlinear calculation and convergence criterion of relative error was chosen as 10^{-7} . The exciting magnetic intensity by current was 1000AT. The large matrix was solved by the Gauss method.

Calculation results of the magnetic flux density distribution on the plates are shown in Fig.6. Maximum magnetic flux density is about 1.58T at both ends of the center plate. Average magnetic flux density in the plate section along A-B-C-D-E-F of the plate is plotted in Fig.7. The maximum difference between calculational and experimental values is 6.4% near the end of the center plate. The average difference is about 1.7%. The magnetic flux distribution in air along G-H is compared with measurements in Fig.8. The calculation results are a little bit higher than the measurement. But, these discrepancies take the same order as those of other calculations such as Nakata and Fujiwara[5].



(a) Magnetization By Conventional Method



(b) Magnetization By Present Method

Fig3 Calculation Results for Case 1

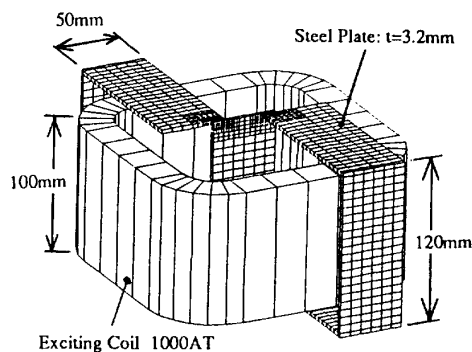


Fig.4 Calculation Model for Team Workshop Problem 13

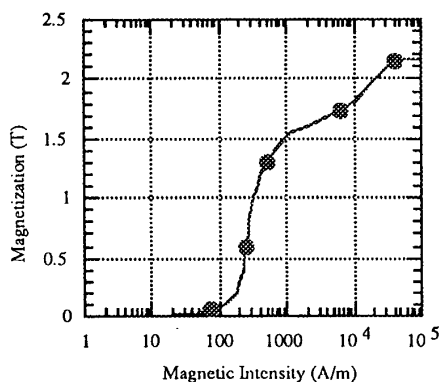


Fig. 5 Magnetization Curve for Steel

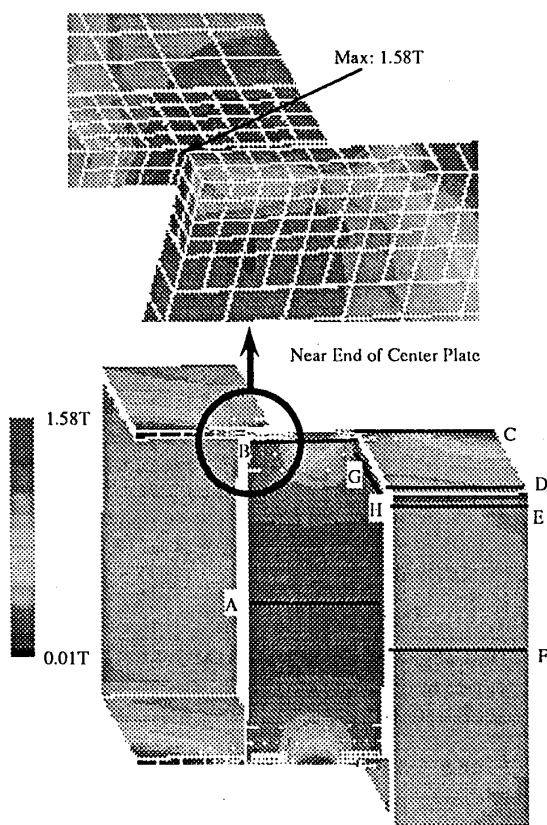


Fig. 6 Magnetic Flux Distribution in Steel Plates

VI. CONCLUDING REMARKS

To calculate nonlinear magnetostatics, a new calculation method based on integral equation method has been developed. The characteristics of the method are as follows: (1) The magnetization vector in an element was expressed by using co- and contra-variant vectors corresponding to each surface of the hexahedron element. (2) The iteration process for nonlinear problems was divided into initialization and convergence parts.

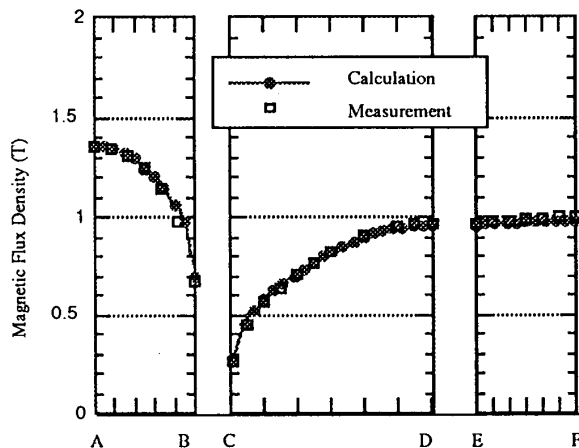


Fig. 7 Average Magnetic Flux Distribution along Plates

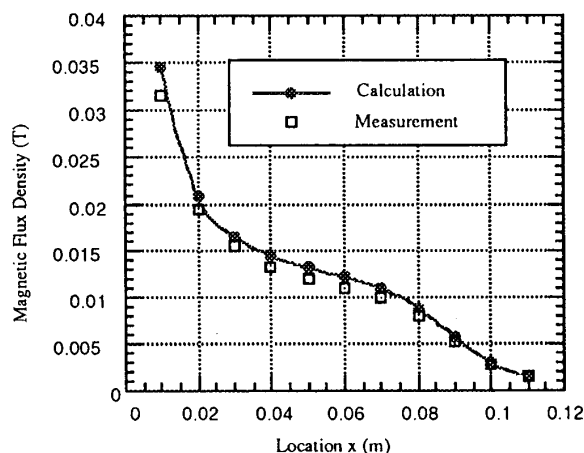


Fig. 8 Magnetic Flux Distribution in Air

The method was applied to the deformed element problem and the TEAM Workshop Problem 13 of the IEEE. (1) Calculation results of the deformed element problem indicated that the method had a smaller effect from element shape than a conventional one. (2) Calculation results of Problem 13 agreed with the measured data within 2%, indicating the method provided good accuracy.

REFERENCES

- [1] M. J. Newman, et al. "GFUN: An Interactive Program as an Aid to Magnet Design", Proc. 4th Magnet Tech. Conf., Brookhaven, 1972.
- [2] Y. Tanaka, "Finite Element Formulation of Magnetic Field Analysis with Integral Equation", Proc. of Combined Technical Meeting on Rotating Machines and Static Apparatus, IEEJ, RM-85-41, SA-85-50, 1985.
- [3] Y. Nojima and A. Ohba "New Constant Element for Magnetic Moment Method", Proc. of 9th Symp. on Electricity and Electronics, Japan Society of Simulation, 1988.
- [4] L. Keutunen and L. R. Turner, "A Volume Integral Formulation for Nonlinear Magnetostatics and Eddy Currents Using Edge Elements", IEEE Trans. on Mag. Vol. 28, No. 2, March 1992.
- [5] T. Nakata et al., "Summary of Results for TEAM Workshop Problem 13 (3-D Nonlinear Magnetostatics Model)", Proc. of Miami TEAM Workshop, 1993.