

Volume integral equation based method for transient scattering from nonlinear penetrable objects—TM case

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1 Introduction

In recent years, electromagnetic scattering from nonlinear objects has been studied using a wide variety of numerical methods, including the finite-difference time-domain (FDTD) method [1], the transmission-line modelling method [2], the finite-element method [3], a frequency-domain/harmonic balance method of moments scheme [4], and other approximate solution techniques [5]. This paper presents a time-domain volume integral equation (TDVIE) based scheme for simulating electromagnetic scattering from nonlinear cylinders. Numerical results obtained using this method are in excellent agreement with data obtained using the FDTD method.

2 Time-Domain Volume Integral Equation

Consider a nonlinear penetrable cylinder that extends along the z -axis. Let S denote the cross-section of the cylinder, and assume that the cylinder is illuminated by a TM_z incident pulse $E^i(\mathbf{r}, t)$. The cylinder is assumed nonmagnetic, i.e., $\mu(\mathbf{r}, t) = \mu_0$, and its permittivity exhibits an instantaneous Kerr-type nonlinearity,

$$\epsilon(\mathbf{r}, t) = (\epsilon^L + \alpha E^2(\mathbf{r}, t)) \epsilon_0 \quad \mathbf{r} \in S \quad (1)$$

where ϵ_0 is the free space permittivity, ϵ^L denotes the linear part of the relative permittivity, α is the coefficient of nonlinearity, and $E(\mathbf{r}, t)$ is the total electric field in the scatterer. This total field is the sum of the incident and scattered fields, i.e.,

$$E(\mathbf{r}, t) = E^i(\mathbf{r}, t) + E^s(\mathbf{r}, t) \quad (2)$$

The scattered electric field $E^s(\mathbf{r}, t)$ may be expressed as

$$E^s(\mathbf{r}, t) = - \left[\mu_0 \int_S ds \frac{\partial^2}{\partial t^2} [(\epsilon(\mathbf{r}', t) - \epsilon_0) E(\mathbf{r}', t)] \otimes g(R, t) \right] \quad (3)$$

where the symbol ' $\otimes$ ' stands for temporal convolution, and $g(R, t)$ is two-dimensional free space Green function,

$$g(R, t) = \frac{1}{2\pi} \frac{u(t - R/c)}{\sqrt{t^2 - (R/c)^2}} \quad (4)$$

here $u(\cdot)$ is the Heaviside step function, c denotes the speed of light in the free space, and $R = |\mathbf{r} - \mathbf{r}'|$. Combining eqs. (2) and (3), yields

$$E(\mathbf{r}, t) + \mu_0 \int_S ds \left[\frac{\partial^2}{\partial t^2} [(\epsilon(\mathbf{r}', t) - \epsilon_0) E(\mathbf{r}', t)] \right] \otimes g(R, t) = E^i(\mathbf{r}, t) \quad (5)$$

which is a nonlinear equation for $E(\mathbf{r}, t)$.

3 Numerical Method

We propose to solve eq. (5) for the electric flux $\mathcal{D}(\mathbf{r}, t) = [\varepsilon(\mathbf{r}, t) - \varepsilon_0]E(\mathbf{r}, t)$ using a marching-on-in-time (MOT) scheme. To this end, $\mathcal{D}(\mathbf{r}, t)$ is expanded in terms of a set of N_s spatial and N_t temporal basis functions, as

$$\mathcal{D}(\mathbf{r}, t) = \sum_{k=1}^{N_s} \sum_{l=1}^{N_t} P_k(\mathbf{r}) Q_k^l(t) \quad (6)$$

The spatial expansion functions $P_k(\mathbf{r})$ are pulse functions defined on a set of N_s rectangular cells S_k , $k = 1, 2, \dots, N_s$, that jointly approximate S , i.e.,

$$P_k(\mathbf{r}) = \begin{cases} 1 & \mathbf{r} \in S_k \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

The functions $Q_k^l(t)$ are quadratic polynomials defined on temporal subintervals of duration Δt , i.e.,

$$Q_k^l(t) = \mathcal{P}_l(t)(a_k^l t^2 + b_k^l t + c_k^l) \quad (8)$$

where

$$\mathcal{P}_l(t) = \begin{cases} 1 & t \in [t_{l1}, t_{l2}] \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

and $t_{l1} = (l-1)\Delta t$, $t_{l2} = l\Delta t$. The quadratic polynomial interpolates the last three time steps, therefore,

$$a_k^l = \frac{\mathcal{D}_k^{l-2} - 2\mathcal{D}_k^{l-1} + \mathcal{D}_k^l}{2\Delta t^2} \quad (10)$$

The other two coefficients are never used because of the second derivative in eq. (5). In eq. (10), $\mathcal{D}_k^l$ is defined as $\mathcal{D}(\mathbf{r}, t)$ at the center of the k^{th} cell at time $t_l = (l-1)\Delta t$. Substituting eq. (6) into eq. (5), and point matching at $\mathbf{r}_m$ yields

$$E(\mathbf{r}_m, t_n) + \frac{\mu_0}{2\pi\Delta t^2} \sum_{k=1}^{N_s} \sum_{l=1}^{N_t} (\mathcal{D}_k^{l-2} - 2\mathcal{D}_k^{l-1} + \mathcal{D}_k^l) V_k^l(\mathbf{r}_m, t_n) = E^i(\mathbf{r}_m, t_n) \quad m = 1, \dots, N_s \quad (11)$$

where

$$V_k^l(\mathbf{r}_m, t_n) = \int_{S_k} ds \left\{ \ln \left[(t_n - t_{l1}) + \sqrt{(t_n - t_{l1})^2 - \left(\frac{R}{c}\right)^2} \right] - \ln \left[(t_n - \tilde{t}_{l2}) + \sqrt{(t_n - \tilde{t}_{l2})^2 - \left(\frac{R}{c}\right)^2} \right] \right\} \quad (12)$$

with $t_{l1} < t - R/c$, $\tilde{t}_{l2} = \min\{t_{l2}, t - R/c\}$, and $t_n = (n-1)\Delta t$, $n = 1, 2, \dots, N_t$.

Recalling the relationship between $\mathcal{D}$ and E , it is clear that eq. (11) is a system of N_s nonlinear equations with N_s unknowns. In this paper, we use the Powell hybrid method [6] to solve this system at each time step.

4 Numerical Results

To validate the TDVIE method, it is applied to the analysis of scattering from the two nonlinear penetrable cylinders shown in Fig. 1. Their electric parameters are

assumed identical : $\epsilon^L = 10.0$ and $\alpha = 8.0 \text{ m}^2/\text{V}^2$. These cylinders are illuminated by the following transient TM_z pulse,

$$E^i(\mathbf{r}, t) = -E_0\sqrt{2\sigma} \left(t - t_0 - \frac{\hat{\mathbf{x}} \cdot \mathbf{r}}{c} \right) \exp \left[-\sigma \left(t - t_0 - \frac{\hat{\mathbf{x}} \cdot \mathbf{r}}{c} \right)^2 \right] \quad (13)$$

where $E_0 = 4.0 \text{ V/m}$, $t_0 = 1.12 \times 10^{-10} \text{ s}$, and $\sigma = 6.22 \times 10^{21} \text{ s}^{-2}$.

We use the TDVIE method to solve the above nonlinear scattering problems, and compare our results to those obtained using the FDTD method. Fig. 2 shows the electric field at point p in the rectangular cylinder, whose size is $l = 100 \text{ }\mu\text{m}$ and $w = 50 \text{ }\mu\text{m}$. The cylinder is divided into 50 cells. These TDVIE and FDTD results agree very well with each other. Fig. 3 shows the results for the concave cylinder. Its length, width and thickness are $l = 810 \text{ }\mu\text{m}$, $w = 210 \text{ }\mu\text{m}$, and $t = 10 \text{ }\mu\text{m}$, respectively. A total of 181 cells are used to discretize the cylinder. Again, these TDVIE and FDTD results agree very well with each other.

This paper described an MOT based TDVIE method for analyzing TM_z scattering from nonlinear cylinders. Numerical examples show that the TDVIE method can be applied to solve the problems of scattering from nonlinear penetrable objects, and provide good results. It should be noted that the present scheme can be accelerated considerably by using the plane-wave time-domain (PWTD) algorithm [7].

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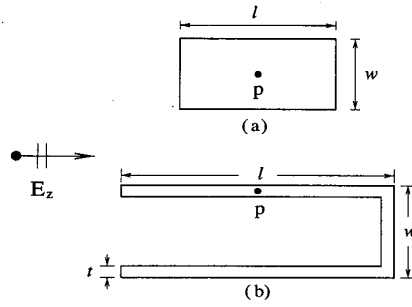


Fig.1 : Three representative penetrable nonlinear cylinders.
(a)square cylinder, and (b)concave cylinder.

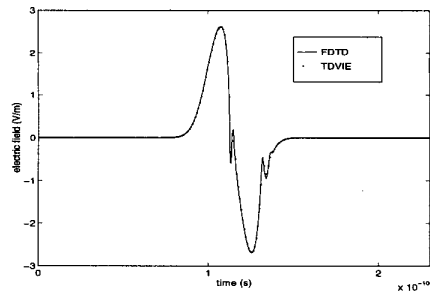


Fig.2 : Comparison of electric fields, at Point p in the rectangular cylinder,
obtained by using the TDVIE and FDTD methods.

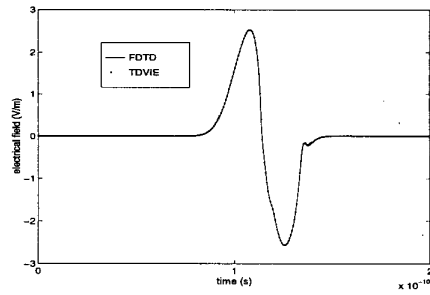


Fig.3 : Comparison of electric fields, at Point p in the concave cylinder,
obtained by using the TDVIE and FDTD methods.