

Fast Transient Analysis of Electromagnetic Scattering from Three-Dimensional Dielectric Inhomogeneities Using a Volume Integral Equation

Noel T. Gres^{†*}, Balasubramaniam Shanker[‡], A. Arif Ergin[†], and Eric Michielssen[†]

[†]Center for Computational Electromagnetics
Dept. of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
1406 W. Green St., Urbana, IL 61801

[‡]Dept. of Electrical and Computer Engineering
Iowa State University
372 Durham, Ames, IA 50010

1. Introduction

A fast technique for analyzing transient electromagnetic scattering from three-dimensional inhomogeneous dielectric targets is proposed. This technique solves a volume integral equation using a marching-on-in-time (MOT) scheme [1]. The computational complexity of performing a volume-scattering analysis using the classical technique in [1] is $O(N_t N_s^2)$, where the volume source density is represented by N_s spatial and N_t temporal samples. This cost is prohibitively high, rendering the method impractical for analyzing scattering from electrically large structures. The contribution of this work is to reduce the high cost of the classical scheme to $O(N_t N_s)$ by augmenting it with the multilevel plane-wave time-domain (PWTD) algorithm [2]. It is shown that incorporation of the PWTD algorithm into the MOT scheme does not compromise the accuracy of the scattering solution.

2. Marching-On-In-Time Scheme

Consider an inhomogeneous scatterer occupying a volume V in an unbounded medium with permittivity ϵ_0 and permeability μ_0 . It is assumed that the scatterer is isotropic, lossless, and nonmagnetic with frequency independent permittivity $\epsilon(\mathbf{r})$. The scatterer is illuminated by an incident electric field $\mathbf{E}^i(\mathbf{r}, t)$ that is bandlimited to ω_{max} and zero on V for time $t < 0$. Our task is to determine the total electric field $\mathbf{E}(\mathbf{r}, t)$ for all space and $t \geq 0$.

It can be shown that the electric flux $\mathbf{D}(\mathbf{r}, t) = \epsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t)$ in V satisfies the integro-differential equation

$$\mathbf{E}(\mathbf{r}, t) - \mathbf{E}^s(\mathbf{r}, t) = \mathbf{E}^i(\mathbf{r}, t), \quad (1)$$

where the scattered electric field $\mathbf{E}^s(\mathbf{r}, t)$ is given by

$$\mathbf{E}^s(\mathbf{r}, t) = -\frac{\mu_0}{4\pi} \partial_t^2 \int_V d\mathbf{v}' \kappa(\mathbf{r}') \frac{\mathbf{D}(\mathbf{r}', \tau)}{R} + \frac{1}{4\pi\epsilon_0} \nabla \int_V d\mathbf{v}' \frac{\nabla' \cdot \kappa(\mathbf{r}') \mathbf{D}(\mathbf{r}', \tau)}{R}. \quad (2)$$

In Equation (2), $\kappa(\mathbf{r}) = (\epsilon(\mathbf{r}) - \epsilon_0)/\epsilon(\mathbf{r})$ is the contrast ratio, $R = |\mathbf{r} - \mathbf{r}'|$ denotes the distance from source to observer, $\tau = t - R/c$ denotes retarded time, and $c = (\mu_0\epsilon_0)^{-1/2}$.

Equation (1) is solved using the classical MOT scheme. To this end, the electric flux density $\mathbf{D}(\mathbf{r}, t)$ is discretized by a set of N_s spatial vector basis functions $\{\mathbf{f}_n(\mathbf{r})\}$ and N_t temporal basis functions $\{T_j(t)\}$ as

$$\mathbf{D}(\mathbf{r}, t) = \sum_{n=1}^{N_s} \sum_{j=0}^{N_t-1} D_{n,j} \mathbf{f}_n(\mathbf{r}) T_j(t), \quad (3)$$

where $\{D_{n,j}\}$ is the set of unknown expansion coefficients. The spatial basis functions used are the volumetric rooftop basis functions defined over tetrahedral elements proposed by Schaubert *et al.* [3]. The temporal basis functions are chosen to be of the form $T_j(t) = T(t - j\Delta t)$, where Δt is the time step size and $T(t)$ is a cubic polynomial interpolation function.

The integral equation is discretized using a Galerkin testing procedure which in matrix form yields

$$\bar{\mathbf{Z}}_0 \mathbf{D}_j = \mathbf{U}_j - \sum_{k=1}^{j-1} \bar{\mathbf{Z}}_k \mathbf{D}_{j-k}. \quad (4)$$

In Equation (4) component m of the vectors $\mathbf{D}_i$ and $\mathbf{U}_i$ is given by $D_{m,i}$ and $\int_V dV \mathbf{f}_m(\mathbf{r}) \mathbf{E}(\mathbf{r}, t_i)$, respectively, and entry mn of the sparse interaction matrix $\bar{\mathbf{Z}}_i$ is the field radiated by element n tested by element m . Equation (4) relates the expansion coefficients at the j th time step to the incident field and the coefficients at prior time steps. Hence, all the expansion coefficients $D_{n,j}$ can be evaluated by starting at the first time step, forming the right-hand side, and solving Equation (4) at each time step. Because of the summation on the right-hand side of Equation (4), which involves the evaluation of the field at each point in V due to all past sources, the computational complexity of the classical MOT scheme is $O(N_s N_t^2)$.

3. Plane-Wave Time-Domain Acceleration

To implement the multilevel PWTD algorithm in the framework of the MOT scheme, the scatterer is enclosed within a cubical box; this box is recursively subdivided into eight child boxes until the finest boxes have linear dimensions proportional to the wavelength at ω_{max} . The smallest boxes are said to reside at level one, while the largest resides at the highest level. Two boxes are in each other's far-field only if their centers are farther apart than a prescribed distance and their parent boxes are not in each other's far-field. Near-field interactions only occur at level one and are computed using the classical MOT scheme; far-field interactions are computed using the PWTD algorithm.

Consider a source and observation box which are in each other's far-field. Their centers are denoted by $\mathbf{r}_s$ and $\mathbf{r}_o$, respectively. In the source box the electric flux $\mathbf{D}_n(\mathbf{r}, t)$ associated with the basis function $\mathbf{f}_n(\mathbf{r})$ is divided into L subsignals each occupying a time interval $[(l-1)T_s, lT_s]$, such that $\mathbf{D}_n(\mathbf{r}, t) = \sum_{l=1}^L \mathbf{D}_{n,l}(\mathbf{r}, t)$, where the subsignal can be separated into its spatial and temporal variations as $\mathbf{D}_{n,l}(\mathbf{r}, t) = \mathbf{f}_n(\mathbf{r}) f_{n,l}(t)$. The field radiated by $\mathbf{D}_{n,l}(\mathbf{r}, t)$ is denoted by $\mathbf{E}_{n,l}^s(\mathbf{r}, t)$. The field tested by $\mathbf{f}_m(\mathbf{r})$ in the observer box can be expressed as

$$\int_V d\nu \mathbf{f}_m(\mathbf{r}) \mathbf{E}_{n,l}^s(\mathbf{r}, t) = \int_{tT_s}^t dt' \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta S_m^-(\hat{\mathbf{k}}, t')^T * T(\hat{\mathbf{k}}, \mathbf{R}_c, t') * S_n^+(\hat{\mathbf{k}}, t') * f_{n,l}(t'), \quad (5)$$

given that the closest distance between source and observer boxes is greater than cT_s and that $t > tT_s$. In Equation (5)

$$T(\hat{\mathbf{k}}, \mathbf{R}_c, t) = -\frac{\mu_0}{8\pi^2 c} \partial_t^4 \delta[t - \hat{\mathbf{k}} \cdot \mathbf{R}_c / c] \quad (6)$$

is known as the translation function,

$$\begin{aligned} S_m^-(\hat{\mathbf{k}}, t) &= \int_V d\nu \hat{\mathbf{k}} \times \mathbf{f}_m(\mathbf{r}) \delta[t - \hat{\mathbf{k}} \cdot (\mathbf{r} - \mathbf{r}_\gamma) / c] \\ S_n^+(\hat{\mathbf{k}}, t) &= \int_V d\nu \kappa(\mathbf{r}) \hat{\mathbf{k}} \times \mathbf{f}_n(\mathbf{r}) \delta[t + \hat{\mathbf{k}} \cdot (\mathbf{r} - \mathbf{r}_\gamma) / c] \end{aligned} \quad (7)$$

are called projection operators, $\hat{\mathbf{k}} = \hat{\mathbf{x}} \sin\theta \cos\phi + \hat{\mathbf{y}} \sin\theta \sin\phi + \hat{\mathbf{z}} \cos\theta$, and $\mathbf{R}_c = \mathbf{r}_\gamma - \mathbf{r}_\gamma$.

The PWTD algorithm accelerates the MOT scheme by replacing Equation (2) with Equation (5) for testing far-fields. This speed-up occurs because Equation (5) can be efficiently evaluated via a three-stage process: (1) forming outgoing rays by performing the right-most convolution, (2) translating outgoing rays from source to observer boxes by performing the center convolution, and (3) superimposing incoming rays onto the observer by performing the left-most convolution. Forming outgoing rays is directly performed only for boxes without children. For boxes with children outgoing rays are constructed using the operations of interpolation and ray splicing [2]. On the other hand, incoming rays for boxes are constructed in two ways: directly by translation and from the parents' incoming rays using the operations of ray resection and antepolation [2]. It can be shown that the volumetric MOT scheme enhanced with this multilevel PWTD algorithm has a computational complexity of $O(N_t N_s)$.

4. Results

This section presents results generated by the PWTD enhanced MOT scheme outlined in the previous sections. The incident field is chosen to be an $\hat{\mathbf{x}}$ polarized modulated Gaussian plane wave propagating in the $\hat{\mathbf{z}}$ direction

$$\mathbf{E}^i(\mathbf{r}, t) = \hat{\mathbf{x}} \cos[(ct - \mathbf{r} \cdot \hat{\mathbf{z}}) \omega_0 / c] e^{-(ct - \mathbf{r} \cdot \hat{\mathbf{z}} - 6\sigma)^2 / 2\sigma^2} \quad (8)$$

where ω_0 is the center frequency and σ is the standard deviation. The effective maximum frequency of the signal is defined as $\omega_{max} = \omega_0 + 6c/\sigma$.

Results are given in terms of the frequency domain radar cross section $RCS(\hat{\mathbf{r}}, \omega)$ given by

$$RCS(\hat{\mathbf{r}}, \omega) = 10 \log_{10} \left[4\pi |\mathbf{E}^{far}(\hat{\mathbf{r}}, \omega)|^2 / |\mathbf{E}^i(\omega)|^2 \right] \quad (9)$$

where $\mathbf{E}^{far}(\hat{\mathbf{r}}, \omega)$ is the Fourier transform of the far-field signature obtained from the

electric flux density generated by the MOT scheme and $E^i(\omega)$ is the magnitude of the Fourier transform of the incident field given by Equation (8).

In order to demonstrate the accuracy of the enhanced scheme, scattering from a dielectric spherical shell of permittivity $2\epsilon_0$ is studied. The inner and outer shell radii measure 0.9 m and 1 m respectively. The parameters chosen for the incident pulse are $\sigma = 2\pi c/\omega_0 = 1.9612$ m. The shell is discretized into 1358 tetrahedra yielding 3060 unknowns. The duration of analysis is 400 time steps where the time step size is 0.33463 ns. First, the backscattered radar cross section $RCS(-\hat{x}, \omega)$ is compared over the entire band of the incident pulse (Figure 1(a)). Next, the radar cross section is examined at a single frequency where the outer shell radius measures 0.925 of a free-space wavelength (Figure 1(b)). The analytical Mie series solution is used for comparison. Good agreement is observed.

Acknowledgements

This research was supported by the Air Force Office for Scientific Research via the MURI program under contract no. F49620-96-1-0025 and by a Department of Energy Computational Science Graduate Fellowship.

References

- [1] N. T. Gres, A. A. Ergin, B. Shanker, and E. Michielssen, "Integral equation based analysis of transient electromagnetic scattering from three-dimensional inhomogeneous dielectric objects," in *16th Annual Review of Progress in Applied Computational Electromagnetics*, 2000.
- [2] A. A. Ergin, B. Shanker, and E. Michielssen, "The plane-wave time-domain algorithm for the fast analysis of transient wave phenomena," *IEEE Antennas and Propag. Mag.*, vol. 41, no. 4, pp. 39-52, Aug. 1999.
- [3] D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," *IEEE Trans. Antennas Propagat.*, vol. 32, no. 1, pp. 77-85, Jan. 1984.

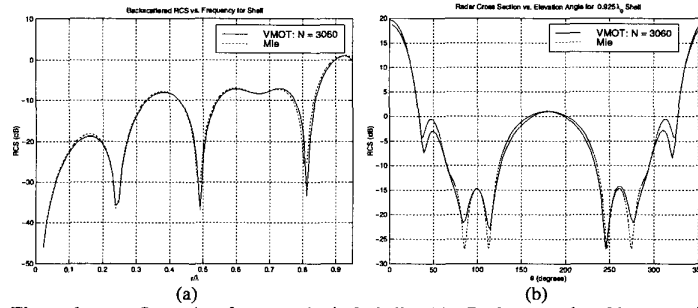


Figure 1. Scattering from a spherical shell. (a) Backscattered RCS vs. r/λ , where r/λ is the outer radius of the shell in wavelengths. (b) RCS vs. elevation angle for $r/\lambda = 0.925$.