

Study of the Convergence of Volume Integral Equation Method

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1. Introduction

To solve the electromagnetic scattering from material objects of arbitrary geometry, a volume integral equation (VIE) formulation is developed [1]. It is shown that perfect conductors can be treated as dielectrics by letting their permittivity approach infinity. In this method, the polarization current is defined by using the equivalence theorem and solved by a moment method. To date the method has been successfully used to solve a number of scattering problems [1], but its convergence has not been studied carefully. This paper addresses this issue and examines the convergence for both dielectric and conductor problems.

2. Formulation of Volume Integral Equation

Assume that a material object is illuminated by a plane wave defined by E^{inc} . A volume integral equation can be obtained by using the equivalence theorem and assuming a volume polarization current J_p flowing inside the objects and generating the scattered field in free space. The boundary condition for the problem can be easily obtained by using the definition of the volume polarization current [2]:

$$E^t = \frac{J_p(r)}{j\omega\epsilon_0(\epsilon_r - 1)} \quad r \in V \quad (1)$$

where ϵ_r is relative complex dielectric constant of the medium, E^t is the total field inside the objects, and the current J_p is polarization current flowing within the objects. Using the free space dyadic Green's function, the concepts of 'principle value' and the source dyadic term in source region, we obtain the following integer-differential equation:

$$E^{inc}(r) + \lim_{\delta \rightarrow 0} \int_{(V-V_\delta)} (J_p(r') \cdot \Psi(r, r')) dV + \frac{\tilde{L} \cdot J_p(r)}{j\omega\epsilon_0} = \frac{J_p(r)}{j\omega\epsilon_0(\epsilon_r - 1)} \quad r \in V \quad (2)$$

where $\Psi(\mathbf{r}, \mathbf{r}')$ is free space dyadic Green's function[2], and $\frac{\tilde{\mathbf{L}} \cdot \mathbf{J}_p(\mathbf{r})}{j\omega\epsilon_0}$ is the source dyadic term used to compensate for the removal of the singularity[3]. To apply a moment method, we assume that the polarization current $\mathbf{J}_p$ has all three components J_{px} , J_{py} , and J_{pz} , and expand this current using pulses as basis functions, that is,

$$J_{pi}(\mathbf{r}) = \sum_{l=1}^L \sum_{m=1}^M \sum_{n=1}^N a_{lmn}^i \cdot P_{lmn}^i(\mathbf{r}) \quad i = x, y, z \quad (3)$$

where i represents x , y or z , a_{lmn}^i are the unknown coefficients and $P_{lmn}(\mathbf{r})$ represent basis functions. Here, they are defined as

$$P_{lmn}^i(\mathbf{r}) = \begin{cases} 1 & \mathbf{r} \in \Delta V_{lmn} \\ 0 & \text{otherwise} \end{cases} \quad i = x, y, z \quad (4)$$

where ΔV_{lmn} is the volume of the small cell lmn . If ΔV_{lmn} are selected small cubical cells, then the equation obtained from boundary condition (2) for the x component is

$$\begin{aligned} -j4\pi\omega\epsilon_0 E_x^{inc}(\mathbf{r}) = & \sum \sum \sum a_{lmn}^x \left(\int_{\Delta V_{lmn}} P_{lmn}^x \left(k_0^2 + \frac{\partial^2}{\partial x^2} \right) G dV - \delta(R) \left(\frac{4\pi}{\epsilon_r - 1} + \frac{4\pi}{3} \right) P_{lmn}^x \right) \\ & + \sum \sum \sum a_{lmn}^y \left(\int_{\Delta V_{lmn}} P_{lmn}^y \frac{\partial^2 G}{\partial x \partial y} dV \right) + \sum \sum \sum a_{lmn}^z \left(\int_{\Delta V_{lmn}} P_{lmn}^z \frac{\partial^2 G}{\partial x \partial z} dV \right) \end{aligned} \quad (5)$$

where

$$\delta(R) = \begin{cases} 0 & \text{if } R = 0 \\ 1 & \text{if } R \neq 0 \end{cases}$$

Other expressions for the y and z components can be found by a permutation of x , y , and z in the above equation. The integrations in equation (5) are principal value and the limit signs were suppressed for the reason of simplicity.

3. The Convergence of VIE method

Here, some examples are provided to show the convergence of this method. The first example is a lossless rectangular dielectric block of dimensions $0.3\lambda \times 0.2\lambda \times 0.1\lambda$, where λ is the free space wavelength. The dielectric constant ϵ_r is (4.0,0.0). The E-plane and H-plane bistatic RCS are calculated and shown in Fig.1. In curve A, the sidelength of the cells is one tenth of the wavelength

inside the dielectric, i.e., $0.1\lambda_d = (0.05)\lambda$. With this cell size, the total number of cells becomes 48. In curve B and C, we use smaller cells and increased the number of cells increases to 162 and 384, respectively. The cube sizes are 0.033λ and 0.025λ , respectively. Their results are compared in Fig.1a and Fig.1b, which show excellent convergence. For lossless dielectric objects of rectangular shapes, VIE method usually gives excellent far field prediction if the cells is about one tenth of the wavelength in the dielectrics.

The second example is a perfectly conducting cube of 0.6λ in size. We now let the permittivity to go to infinity and calculate E-plane and H-plane bistatic RCS. They are shown in Fig.2. In solving the conductor problems it was found that it is sufficient to use the outer most cells, plus a single point at the centre of the object. To examine the convergence, the cell sizes of 0.1λ , 0.08λ , 0.07λ , and 0.06λ are selected, that results in 219, 297, 387 and 489 cells, respectively. The results show that, for a perfectly conducting cube, or a rectangular block, cell size of 0.1λ again give satisfactory results, but for better prediction of nulls 0.08λ cell size should be selected.

The third example is a combination of lossy rectangular dielectric rod and a perfectly conducting square plate as shown in Fig.3. The dielectric rod size is $0.1\lambda \times 0.1\lambda \times 0.4\lambda$ and has a complex permittivity $\epsilon_r = (10., -0.1)$. The conducting plate is $0.5\lambda \times 0.5\lambda \times 0.1\lambda$. The incidence angles are $\theta_0 = 45^\circ$ and $\phi_0 = 45^\circ$. The calculated E-plane and H-plane bistatic RCS are shown in Fig.3. For curve A, B and C, the cell sizes in conductor and dielectric section are $(0.1\lambda, 0.033\lambda)$, $(0.1\lambda, 0.025\lambda)$, and $(0.05\lambda, 0.025\lambda)$, that resulted in 133, 281 and 456, respectively. They show that the convergence is achieved for curves B and C, with the smaller cell sizes than previous examples due to the complexity of the geometry.

In conclusion, VIE method generally converges rapidly as long as the segmentation and cells approximate the object well. Generally speaking, for dielectric cubes or rectangular cylinders, to get good far field prediction, the maximum size of cells must be about one tenth of the wavelength in the dielectrics. For conducting object, the size must be about one tenth of the wavelength in free space. For more complicated objects like example 3, smaller cells are required.

References:

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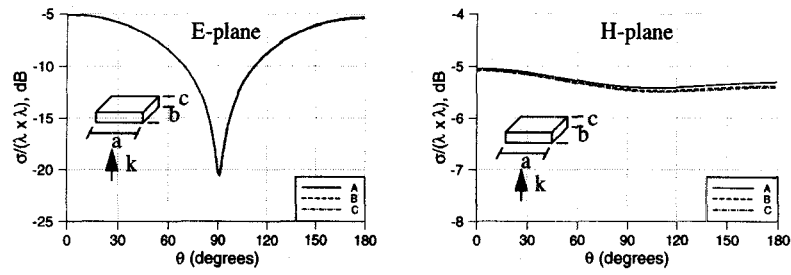


Fig.1 Bistatic RCS of a lossless dielectric rectangular block with different cell sizes

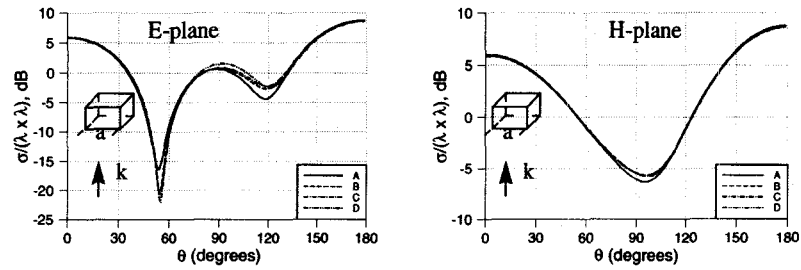


Fig.2 Bistatic RCS of a perfectly conducting cube of 0.6λ in size with different cell sizes

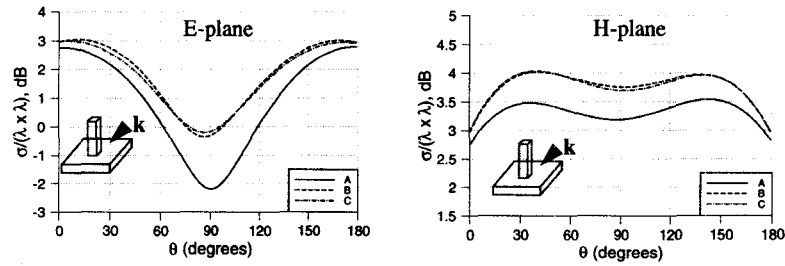


Fig.3 Bistatic RCS of a combination of dielectric and conductor objects shown in the figure, with different cell size