

AN ALGORITHM OF THE METHOD OF VOLUME INTEGRAL EQUATIONS FOR ANALYSIS OF WAVEGUIDE ARRAYS WITH PROTRUDING DIELECTRIC ELEMENTS

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Abstract: An algorithm for calculating characteristics of parallel-plate waveguide arrays with layered dielectric filling and protruding dielectric elements when exciting the waveguides by *TEM*-waves is proposed. The algorithm is based on the method of volume integral equations for the electric field in the protrusions, and on the method of projection matching the fields on the waveguide aperture. The obtained equations are solved by the method of moments with using the piecewise-constant approximation of the field in the protrusion in the longitudinal direction and cosine expansion in each cross section. Results of calculation for a number of structures are presented.

Owing to a number of attractive features of the waveguides with protruding dielectric elements the latter are often used in phased array antennas. However, although some methods for analysis of the indicated arrays have been proposed in literature, e.g. [1–3], their capabilities are limited, and, for this reason, creation of new efficient algorithms in the field in question remains a problem of importance.

The present paper describes an algorithm realized for the case of an infinite periodic array of parallel-plate waveguides with layered dielectric filling and protruding dielectric elements of arbitrary shape. The waveguides are assumed to be excited by *TEM*-waves with complex amplitudes $\exp(is'u)$, where s' is a guide number and u is a phase shift.

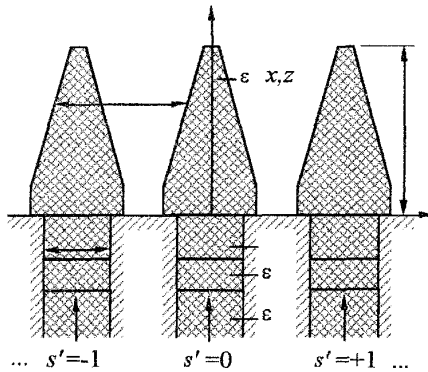


Fig. 1. Geometry of array.

Calculation of the field reflected back into the waveguides and the field radiated into free space is carried out with using the concept of polarization currents for the dielectric structure above the waveguides $j^p(x, z) = -i\omega\epsilon_0[\epsilon_1(x, z) - 1]E(x, z)$, where $\epsilon_1(x, z)$ is a distribution of relative permittivity in a protrusion, and $E(x, z)$ is intensity of the electric field in the dielectric (the omitted time dependence is taken in the form $\exp(-i\omega t)$).

The field above the waveguides is written as a sum of waves going from the waveguide apertures and waves radiated by the polarization currents:

$$H_y(x, z) = \frac{i\omega\epsilon_0}{k} \sum_{q=-\infty}^{\infty} D_q e^{i\alpha_q x + \Gamma_q z} + \text{rot}_y \tilde{A}(x, z), \quad (1)$$

$$E(x, z) = -\text{rot} \frac{\hat{y}}{k} \sum_{q=-\infty}^{\infty} D_q e^{i\alpha_q x + \Gamma_q z} - \frac{1}{i\omega\epsilon_0} (\text{grad div} + k^2) \tilde{A}(x, z), \quad (2)$$

where

$$\tilde{A}(x, z) = \frac{i\omega\epsilon_0}{2b} \int_0^b dz' \int_{-t(z')/2}^{t(z')/2} dx' [\epsilon_1(x', z') - 1] E(x', z') \sum_{q=-\infty}^{\infty} \frac{e^{\Gamma_q |z-z'|} - g_q(z, z')}{\Gamma_q} e^{i\alpha_q (x-x')} \quad (3)$$

is a changed expression for the potential created by the polarization currents,

$$g_q(z, z') = \exp[\Gamma_q(z - z')],$$

$$D_q = C_q + \frac{k}{2b} \int_0^h dz' \int_{-t(z')/2}^{t(z')/2} dx' [\varepsilon_1(x', z') - 1] [E_x(x', z') - i\alpha_q E_z(x', z') / \Gamma_q] e^{-i\alpha_q x' - \Gamma_q z'}$$

for propagating Floquet waves as well as for evanescent waves with $|\Gamma_q|$ close to zero; $g_q(x, z) = 0$ and $D_q = C_q$ for the rest waves, C_q are amplitudes of the waves going up from the apertures, $\Gamma_q = i[k^2 - \alpha_q^2]^{1/2}$ and $\alpha_q = (u + 2\pi q)/b$ are propagation constants, and $t(z')$ is the width of the protrusion.

The electric field to be determined in the protrusion is represented in the form

$$E(x, z) = \sum_{l=1}^L \Omega_l(z) \sum_{v=0}^{\infty} E_{lv} \cos \frac{v\pi(x + t_l/2)}{t_l}, \quad (4)$$

$t_l = t(z_l)$, $z_l = (2l-1)\Delta$, $\Delta = h/(2L)$, $\Omega_l(z) = 1$ if $|z - z_l| < \Delta$ and $\Omega_l(z) = 0$ otherwise.

The projection matching of fields (1) and (2) to the fields in the waveguide on the waveguide aperture with taking into account (4) gives the following equations:

$$\begin{aligned} \frac{\xi_m^+ B_m}{2 - \delta_{0m}} - \sum_{q=-\infty}^{\infty} D_q Q_{mq}^* + \sum_{l=1}^L \frac{kt_l}{2b} \sum_{v=0}^{\infty} \left\{ -E_{xlv} \sum_{s=-\infty}^{\infty} \frac{\zeta'_{ls}(0) - \tilde{\zeta}'_{ls}(0)}{\Gamma_s} Q_{ms}^* \tilde{P}_{lvs} + \right. \\ \left. + E_{zlv} \sum_{s=-\infty}^{\infty} i\alpha_s \frac{\zeta_{ls}(0) - \tilde{\zeta}_{ls}(0)}{\Gamma_s} Q_{ms}^* \tilde{P}_{lvs} \right\} = -\eta_0 \delta_{0m}, \quad m = 0, 1, \dots, \end{aligned} \quad (5)$$

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{\gamma_{n2} \alpha}{\varepsilon_2 k b} \xi_n Q_{np} B_n + \frac{\Gamma_p}{k} D_p + \sum_{l=1}^L \frac{kt_l}{2b} \sum_{v=0}^{\infty} \left\{ E_{xlv} \Gamma_p [\zeta_{lp}(0) - \tilde{\zeta}_{lp}(0)] + \right. \\ \left. + E_{zlv} i\alpha_p [\zeta'_{lp}(0) - \tilde{\zeta}'_{lp}(0)] / \Gamma_p \right\} \tilde{P}_{lvp} = \frac{\gamma_{02} \alpha}{\varepsilon_2 k b} Q_{0p} \eta_0, \quad p = 0, \pm 1, \dots, \end{aligned} \quad (6)$$

where η_0 and $\xi_n^\pm$ are coefficients determined by the scattering matrix of the intermediate layer in the waveguide [3], B_n and $\gamma_{n2} = i[k^2 \varepsilon_2 - (n\pi/a)^2]^{1/2}$ are amplitudes of the reflected waves and propagation constants, respectively, in the upper layer in the waveguide,

$$\begin{aligned} Q_{mq} = \frac{1}{a} \int_{-a/2}^{a/2} \cos \frac{m\pi(x + a/2)}{a} e^{-i\alpha_q x} dx, \quad \tilde{P}_{lvq} = \frac{1}{t_l} \int_{-t_l/2}^{t_l/2} [\varepsilon_1(x, z_l) - 1] \cos \frac{v\pi(x + t_l/2)}{t_l} e^{-i\alpha_q x} dx, \\ \zeta_{ls}(z) = \int_{z_l-\Delta}^{z_l+\Delta} e^{\Gamma_s |z-z'|} dz', \quad \tilde{\zeta}_{ls}(z) = \int_{z_l-\Delta}^{z_l+\Delta} g_s(z, z') dz'. \end{aligned}$$

Application of the method of moments to integral equation (2) for the electric field in the protrusion complements the algebraic system (5) and (6) by two more parts:

$$\begin{aligned} - \sum_{q=-\infty}^{\infty} \frac{\Gamma_q}{k} e^{\Gamma_q z_l} P_{l'v'q}^* D_q + \sum_{l=1}^L \sum_{v=0}^{\infty} \left\{ E_{xlv} \left[\frac{\delta_{l'v} \delta_{vv}}{2 - \delta_{vv}} - \frac{kt_l}{2b} \sum_{s=-\infty}^{\infty} \Gamma_s [\zeta_{ls}(z_{l'}) - \tilde{\zeta}_{ls}(z_{l'})] P_{l'v's}^* \tilde{P}_{lvs} \right] + \right. \\ \left. + E_{zlv} \frac{kt_l}{2b} \sum_{s=-\infty}^{\infty} i\alpha_s \frac{\zeta'_{ls}(z_{l'}) - \tilde{\zeta}'_{ls}(z_{l'})}{\Gamma_s} P_{l'v's}^* \tilde{P}_{lvs} \right\} = 0, \end{aligned} \quad (7)$$

$$\sum_{q=-\infty}^{\infty} \frac{i\alpha_q}{k} e^{\Gamma_q z_{l'}} P_{l'v'q}^* D_q + \sum_{l=1}^L \sum_{v=0}^{\infty} \left\{ E_{xlv} \frac{kt_l}{2b} \sum_{s=-\infty}^{\infty} i\alpha_s \frac{\zeta'_{ls}(z_{l'}) - \tilde{\zeta}'_{ls}(z_{l'})}{\Gamma_s} P_{l'v's}^* \tilde{P}_{lvs} + E_{zlv} \left[\frac{\delta_{l'l'} \delta_{v'v}}{2 - \delta_{v'0}} + \right. \right. \\ \left. \left. + \frac{kt_l}{2b} \sum_{s=-\infty}^{\infty} \frac{\zeta''_{ls}(z_{l'}) - \tilde{\zeta}''_{ls}(z_{l'}) + k^2 [\zeta_{ls}(z_{l'}) - \tilde{\zeta}_{ls}(z_{l'})]}{\Gamma_s} P_{l'v's}^* \tilde{P}_{lvs} \right] \right\} = 0, \quad l'=1, \dots, L, \\ v'=0, 1, \dots, \quad (8)$$

where P_{lvs} is calculated in the same way as $\tilde{P}_{lvs}$ but without factor $[\varepsilon_1(x, z_l) - 1]$.

After solving the truncated system (5)–(8), the array reflection coefficient is determined by B_0 and the corresponding elements of the scattering matrix of the intermediate layer [3], and the amplitude element pattern corresponding to unit incident power is determined as $F(\theta) = (\varepsilon_4^{-1/2} b/a)^{1/2} |D_0(kb \sin \theta)| \cos \theta$.

The algorithm described above has been realized in *Fortran* and *C++* codes, and tested with using the results available in [1], [3], and [4]. Some examples of calculated element patterns for arrays of relatively small and large spacing are shown in Figs. 2 and 3. The patterns in Fig. 2 correspond to $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 2.56$, $h = 0.5\lambda$, $t(z) = a$ (solid curve), and $t(z) = a(1 - z/h)$ (dashed). The patterns in Fig. 3 have been obtained for $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 2$, $h = 1.2\lambda$, $t(z) = 0.6\lambda$ (solid curve), and $t(z) = 0.6\lambda$ for $z < 0.1\lambda$, $t(z) = (7.1\lambda - 5z)/11$ for $0.1\lambda \leq z \leq 1.2\lambda$ (dashed). The orders of the algebraic systems for the presented examples were 120 and 250, that allows to consider the proposed approach as a promising one for generalizing it for a 3D case.

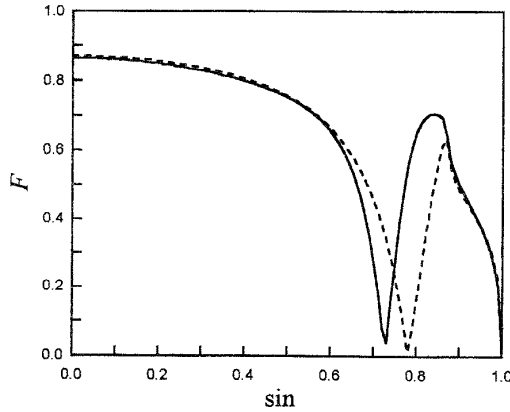


Fig. 2. Array with $a = 0.23112\lambda$, $b = 0.535\lambda$.

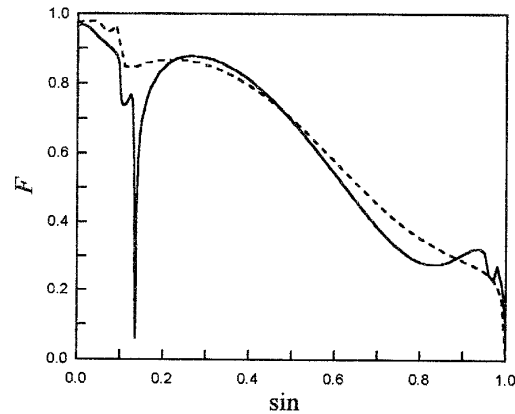


Fig. 3. Array with $a = 0.3\lambda$, $b = 0.91\lambda$.

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