

A Fast Volume Integral Equation Solver for Electromagnetic Scattering From Large Inhomogeneous Objects in Planarly Layered Media

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Abstract—A newly developed iterative method, the stabilized biconjugate gradient fast Fourier transform (BCGS-FFT) method is applied to simulate electromagnetic scattering from large inhomogeneous objects embedded in a planarly layered medium. In this fast solver, the weak-form formulation is applied to obtain a less singular discretization of the volume electric field integral equation. Several techniques are utilized to speed up the dyadic Green's function evaluation. To accelerate the operation of the dyadic Green's function on an induced current (i.e., the "Green's operation"), the Green's function is split into convolutional and correlational components so that FFT can be applied. The CPU time and memory cost of this BCGS-FFT method is $O(N \log N)$ and $O(N)$, respectively, where N is the number of unknowns, significantly more efficient than the method of moments (MoM). As a result, this method is capable of solving large-scale electromagnetic scattering problems in a planarly layered background. A large-scale scattering problem in a layered medium with more than three million unknowns has been solved on a Sun Ultra 60 workstation with 1.2 GBytes memory.

Index Terms—Stabilized biconjugate gradient fast Fourier transform (BCGS-FFT) method, fast algorithm, layered media, scattering, volume integral equation.

I. INTRODUCTION

ELECTROMAGNETIC scattering problems are important to many applications. Many techniques have been developed to stimulate antennas and scatterers residing in a homogeneous space. Although the assumption of a homogeneous background gives reasonable approximate solution in some aerospace applications where the effect of the ground surface can be neglected, it is not applicable to many practical situations where the proximity of the earth must be taken into account. In fact, in many cases the influence of the ground or the ocean, which often can be adequately characterized by a model of one or more planar, dielectric layers, is the dominant effect in the problem. This scattering model is depicted in Fig. 1. The background is a planarly layered structure with M layers. Each layer is homogeneous; it has infinite dimensions in the x and y directions but finite (or semi-infinite for layers 1

and N) in z direction. The electrical properties of each layer are characterized by constant permittivity ϵ_i , permeability μ_i and conductivity σ_i ($i = 1, \dots, M$). The scattering object is characterized by space dependent dielectric constant $\epsilon_r(\mathbf{r})$ and electric conductivity $\sigma(\mathbf{r})$. This model can represent an oil reservoir, utility pipes, buried land mines, or unexploded ordnance (UXO). To the authors' knowledge, there is no existing fast solver for electromagnetic scattering from inhomogeneous objects in layered media. Therefore, this work focuses on developing a fast numerical method for electromagnetic scattering from inhomogeneous objects in a planarly layered structure.

Because the scattering object has an arbitrary inhomogeneity, surface integral equation methods [1]–[3] are in general not appropriate for this application. For general inhomogeneous objects, the volume integral equation methods are more suitable. To solve the electric field integral equation (EFIE) for electromagnetic scattering from inhomogeneous objects, the straightforward solution is the method of moments (MoM). One class of fast methods involve an iterative solver combined with the fast Fourier transform (FFT) algorithm. Early work in this area includes the conjugate gradient (CG) FFT method, the biconjugate gradient (BCG) FFT method and the stabilized biconjugate gradient (BCGS) FFT method in [4]–[10] for objects in a homogeneous background. It has been shown that two different BCG-FFT methods are more efficient than the CG-FFT method for several examples from electromagnetic scattering in a homogeneous background [7], [8]. In [10], the BCGS-FFT method has been applied to solve large-scale scattering problems in a homogeneous background accurately and more efficiently than the CG- and BCG-FFT methods.

However, these FFT accelerated methods have been developed mainly for objects in a homogeneous background, and only recently fast algorithms have been developed for layered media. The CG-FFT method has also been applied in a half-space background by Cui and Chew in [11]. Recently, the BCGS-FFT method has been developed and validated in a letter [12] for electromagnetic scattering in general planarly layered media. This present paper is distinct from [12] in that 1) the BCGS-FFT method is described in detail, and 2) its applications in solving plane wave scattering problems involving large-scale inhomogeneous objects in planarly layered media are presented. In contrast, [12] only gives a brief summary of the method, and only a small problem with a dipole source is presented.

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The main ingredients of the BCGS-FFT method for a layered medium includes 1) the splitting of the dyadic Green's function for the layered medium; 2) fast Green's function evaluation; 3) FFT acceleration using the convolution and correlation theorems; and 4) the stabilized biconjugate-gradient (BiCGSTAB) method for the linear system. The computer memory and CPU time requirements of this method are $O(N)$ and $O(N \log N)$, respectively, where N is the number of unknowns within the inhomogeneous target. This method is thus highly competitive. Indeed, even on a Sun Ultra 60 workstation with 1.2 GBytes memory, this method can solve a large-scale problem with more than three million unknowns.

This paper is organized as follows. In Section II, the volume EFIE is formulated and discretized. In Section III, the BCGS-FFT method is further validated and the RCS of several large size targets, such as underground bunkers and buried UXO objects in layered media, are presented.

II. THEORY

The geometry of the electromagnetic scattering problem is depicted in Fig. 1, where the object is inhomogeneous, while the background medium has M planar layers extending to infinity in x and y directions. The layered medium has a piecewise constant profile of independent permittivity ϵ_m , conductivity σ_m , and permeability μ_m ($m = 1, \dots, M$). The complex permittivity is defined as $\tilde{\epsilon}_m = \epsilon_m + \sigma_m/j\omega$. We will assume that the inhomogeneous object is completely embedded in layer i (possibly with its outer boundary touching the top and/or bottom boundaries in layer i). Furthermore, we will assume that the object has a constant permeability $\mu = \mu_i$ so that there is no contrast in μ between the object and layer i . The objective of this work is to compute the electromagnetic fields in this complex medium. We will focus on the scattering due to a plane wave in this work.

A. EFIE

In a planarly layered medium shown in Fig. 1, an inhomogeneous dielectric object of arbitrary shape is completely embedded in the i th layer. In the m th layer, The total electric field $\mathbf{E}_m(\mathbf{r})$ is the sum of the incident field $\mathbf{E}_m^{\text{inc}}(\mathbf{r})$ and the scattered field $\mathbf{E}_m^s(\mathbf{r})$, or

$$\mathbf{E}_m^{\text{inc}}(\mathbf{r}) = \mathbf{E}_m(\mathbf{r}) - \mathbf{E}_m^s(\mathbf{r}) \quad (1)$$

where $\mathbf{r}$ is the position vector in the three-dimensional space, and $\mathbf{E}_m^{\text{inc}}$ is the electric field in the absence of the object but with the presence of the layered medium. The scattered field in the m th layer can be expressed in terms of the magnetic vector potential and scalar potential as

$$-\mathbf{E}_m^s(\mathbf{r}) = j\omega \mathbf{A}^{mi}(\mathbf{r}) + \nabla \phi^{mi}(\mathbf{r}) \quad (2)$$

where $\mathbf{A}^{mi}(\mathbf{r})$ is the magnetic vector potential in the m th layer due to the induced current source $\mathbf{J}$ inside the inhomogeneous object embedded in the i th layer. This magnetic vector potential is given by

$$\mathbf{A}^{mi}(\mathbf{r}) = \mu_m \int_V \mathbf{G}^{mi}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\mathbf{r}'$$

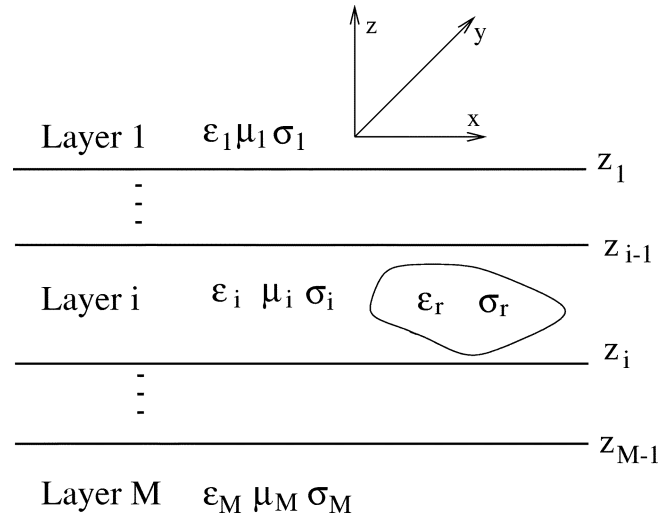


Fig. 1. Typical geometry for electromagnetic scattering in a planarly layered medium.

$$= j\omega \mu_m \int_V \mathbf{G}^{mi}(\mathbf{r}, \mathbf{r}') \cdot \chi(\mathbf{r}') \mathbf{D}(\mathbf{r}') d\mathbf{r}' \quad (3)$$

where $\mathbf{G}^{mi}$ is the dyadic Green's function to be discussed below, V is the volume of the scattering object, $\chi(\mathbf{r}) = \tilde{\epsilon}(\mathbf{r}) - \tilde{\epsilon}_i/\tilde{\epsilon}(\mathbf{r})$ is the contrast function, $\tilde{\epsilon}(\mathbf{r}) = \epsilon(\mathbf{r}) + \sigma(\mathbf{r})/j\omega$, $\mathbf{D}(\mathbf{r}) = \tilde{\epsilon}(\mathbf{r})\mathbf{E}(\mathbf{r})$ is the electric flux density, and $\tilde{\epsilon}_i$ is the complex permittivity of the i th layer. The scalar potential $\phi^{mi}(\mathbf{r})$ is related to $\mathbf{A}^{mi}(\mathbf{r})$ through the Lorenz condition

$$\phi^{mi}(\mathbf{r}) = \frac{j\omega}{k_m^2} \nabla \cdot \mathbf{A}^{mi}(\mathbf{r}) \quad (4)$$

where $k_m = \omega\sqrt{\tilde{\epsilon}_m\mu_m}$ is the wavenumber of the m th layer.

Substituting (2), (3), and (4) into (1) yields

$$\mathbf{E}_m^{\text{inc}}(\mathbf{r}) = \frac{\mathbf{D}_m(\mathbf{r})}{\tilde{\epsilon}_m} - (k_m^2 + \nabla \nabla \cdot) \frac{1}{\tilde{\epsilon}_m} \int_V \mathbf{G}^{mi}(\mathbf{r}, \mathbf{r}') \cdot \chi(\mathbf{r}') \mathbf{D}_i(\mathbf{r}') d\mathbf{r}'. \quad (5)$$

When $m = i$ and $\mathbf{r} \in V$, the above equation can be rewritten as

$$\mathcal{L}[\mathbf{D}_i(\mathbf{r})] = \mathbf{E}_i^{\text{inc}}(\mathbf{r}) \quad (6)$$

where $\mathcal{L}$ is a linear operator defined as

$$\mathcal{L}[\mathbf{r}] = \frac{[\mathbf{r}]}{\tilde{\epsilon}_i} - (k_i^2 + \nabla \nabla \cdot) \frac{1}{\tilde{\epsilon}_i} \int_V \mathbf{G}^{ii}(\mathbf{r}, \mathbf{r}') \cdot \chi[\mathbf{r}'] d\mathbf{r}', \quad \mathbf{r} \in V. \quad (7)$$

Solving (6), the electric flux density $\mathbf{D}_i(\mathbf{r})$ is obtained for $\mathbf{r} \in V$. The relationship $\mathbf{J} = j\omega\chi\mathbf{D}$ can be used to calculate the induced current inside the scattering object, and thus the electric field at any location can be easily obtained from (2).

The volume EFIE has been formulated with the electric flux density as unknowns because the continuity of the normal component of electric flux density can be easily enforced by choosing the roof-top basis functions for $\mathbf{D}$. On the other hand, if the electric/magnetic fields are chosen as the unknowns,

implementation of the boundary conditions is not trivial. Furthermore, the “weak-form” discretization process proposed by Zwamborn and van den Berg [5] for the scattering of objects in a homogeneous background can be extended to layered media to reduce the singularity of the problem.

B. Dyadic Green’s Function for a Layered Medium

In (3), $\mathbf{G}^{mi}$ is the dyadic Green’s function of the layered medium. It represents the magnetic vector potential in layer m due to a unit strength electric dipole in layer i ; the three columns of $\mathbf{G}^{mi}$ correspond to the magnetic vector potential of the dipole oriented in x , y , and z directions, respectively. It can be found by solving the inhomogeneous vector Helmholtz equation

$$(\nabla^2 + k_m^2) \mathbf{G}^{mi}(\mathbf{r}, \mathbf{r}') = -\mathbf{I}\delta(\mathbf{r} - \mathbf{r}') \quad (8)$$

where $\mathbf{I}$ is the unit dyadic. The Green’s function is subject to the condition that the tangential components of the scattered electric field and magnetic field should be continuous across the interfaces between the dielectric layers. It is known that [13] two components of the magnetic vector potential are required to satisfy the boundary conditions at the layer interfaces. In this paper, the “traditional” form [1] is chosen for the dyadic Green’s function

$$\mathbf{G}^{mi}(\mathbf{r}, \mathbf{r}') = (\hat{x}\hat{x} + \hat{y}\hat{y})G_{xx}^{mi} + \hat{x}\hat{z}G_{zx}^{mi} + \hat{z}\hat{y}G_{zy}^{mi} + \hat{z}\hat{z}G_{zz}^{mi} \quad (9)$$

in the spatial domain. The closed form of the Green’s function in spectral domain can be derived by using the equivalent transmission line network. The Green’s function in spatial domain is obtained from its expression in spectral domain by performing inverse Fourier transform in the x and y directions, which is equivalent to Sommerfeld integral evaluation. For example, if $\bar{\rho} = \hat{x}x + \hat{y}y$,

$$G_{xx}^{mi}(|\bar{\rho} - \bar{\rho}'|, z, z') = S_0 \left\{ \tilde{G}_{xx}^{mi}(k_\rho, z, z') \right\} \quad (10)$$

where $S_n[\cdot]$ denotes the Sommerfeld integral (Hankel transform) of order n

$$S_n \left\{ \tilde{f}(k_\rho) \right\} = \frac{1}{2\pi} \int_0^\infty \tilde{f}(k_\rho) J_n(k_\rho |\bar{\rho} - \bar{\rho}'|) k_\rho^{n+1} dk_\rho. \quad (11)$$

The explicit expressions for the dyadic Green’s function in spectral and spatial domains can be found in [3].

C. Splitting of the Dyadic Green’s Function

When solving the integral equation in (6) iteratively, we need to calculate the interactions of the dyadic Green’s function $\mathbf{G}^{ii}(\mathbf{r}, \mathbf{r}')$ with the induced current density $\mathbf{J} = j\omega\chi\mathbf{D}$ for every testing function within the object. For simplicity, we can call this the “Green’s operation.” If the object is discretized into N unknowns, the straightforward computation of these interactions requires $O(N^2)$ arithmetic operations. In the free space, this computation can be accelerated by using the FFT algorithm because the Green’s function there is shift invariant.

As a result of the inhomogeneity along z direction, however, the dyadic Green’s function for a layered medium is no longer

shift invariant, thus the integral kernel of the volume EFIE (6) is not convolutional. For example, in the spectral domain

$$\begin{aligned} \tilde{G}_{xx}^{ii}(k_\rho, z, z') &= \frac{1}{2jk_{z,i}} \left[e^{-jk_{z,i}|z-z'|} + \frac{1}{D_i^{TE}} \right. \\ &\times \left\{ \overleftarrow{R}_i^{TE} e^{-jk_{z,i}[(z+z')-2z_i]} + \overrightarrow{R}_{i-1}^{TE} e^{-jk_{z,i}[2z_{i-1}-(z+z')]} \right. \\ &\left. \left. + 2\overleftarrow{R}_i^{TE} \overrightarrow{R}_{i-1}^{TE} e^{-j2\psi_i} \times \cos k_{z,i}(z-z') \right\} \right] \end{aligned} \quad (12)$$

where $\overleftarrow{R}_i^{TE}$ and $\overrightarrow{R}_{i-1}^{TE}$ define some global reflection coefficients, $\psi_i = k_{z,i}(z_{i-1} - z_i)$, and $D_i^{TE} = 1 - \overleftarrow{R}_i^{TE} \overrightarrow{R}_{i-1}^{TE} e^{-j2\psi_i}$ (see [1], [2]). Obviously, this Green’s function is not shift-invariant in z direction, even though it is still shift-invariant in x and y directions. Consequently, unlike the scattering problem in the free space, the integral in (7) is no longer convolutional; thus, the convolutional theorem cannot be used directly to arrive at the FFT acceleration in the CG-FFT, BCG-FFT, and BCGS-FFT methods as in the case of a homogeneous background.

Fortunately, we are able to develop a splitting technique to rewrite the dyadic Green’s function into two terms, one is shift-invariant in all directions, the other is shift-invariant in the x and y directions, but as a function of $z + z'$ in the z direction. More specifically, as indicated in (12), the dyadic Green’s function in a layered medium can be written as the summation of two terms:

$$\mathbf{G}^{ii}(\mathbf{r}, \mathbf{r}') = \mathbf{G}_1^{ii}(|\bar{\rho} - \bar{\rho}'|, z - z') + \mathbf{G}_2^{ii}(|\bar{\rho} - \bar{\rho}'|, z + z'). \quad (13)$$

As a result, in (7), the interaction between $\mathbf{G}_1^{ii}(\mathbf{r}, \mathbf{r}')$ and $\chi(\mathbf{r}')\mathbf{D}(\mathbf{r}')$ is a convolution in all (x, y, z) directions, and that between $\mathbf{G}_2^{ii}(\mathbf{r}, \mathbf{r}')$ and $\chi(\mathbf{r}')\mathbf{D}(\mathbf{r}')$ is a convolution in x and y directions, but a correlation in z direction. In other words, the integral kernel in the EFIE is convolutional in the x and y directions and convolutional or correlational in the z direction after the splitting. Consequently, the FFT can be applied to these two operations separately through the convolution and correlation theorems, reducing CPU time of the “Green’s operation” from $O(N^2)$ to $O(N \log N)$.

D. Acceleration of Dyadic Green’s Function Evaluation

Another computational burden for the scattering problem is the calculation of the dyadic Green’s function within the volume of the scattering object, which can be a bottleneck for the volume integral equation method. Previous acceleration techniques for the evaluation of this dyadic Green’s function include the complex image method [14], [15], the steepest descent path integration [16], and filter integration method [17]. The first two techniques can become quite involved for a multilayer medium, while the filter technique is more advantageous when dealing with near-field points. In this work, two techniques are applied to reduce the computational cost of the Green’s function.

Singularity Subtraction: First, the Green’s function in the spectral domain can be highly oscillatory as a function of k_ρ , especially when the source and observation planes are close (i.e., z is close to z'). This makes the evaluation of the Green’s function in the spatial domain slow because a large number of quadrature

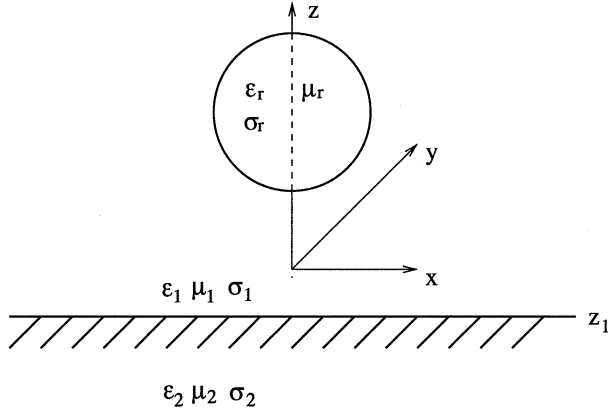


Fig. 2. Configuration of a sphere (radius 0.3 m, $\epsilon_r = 3.0$, $\sigma = 0.01$ S/m) centered at (0, 0, 0.7) m in a half space. The top layer is air, while the ground has $\epsilon_{r2} = 5.0 - 0.2j$, $\mu_{r2} = 1.0$, $\sigma_2 = 0$ S/m; the ground surface is at $z_1 = 0$ m.

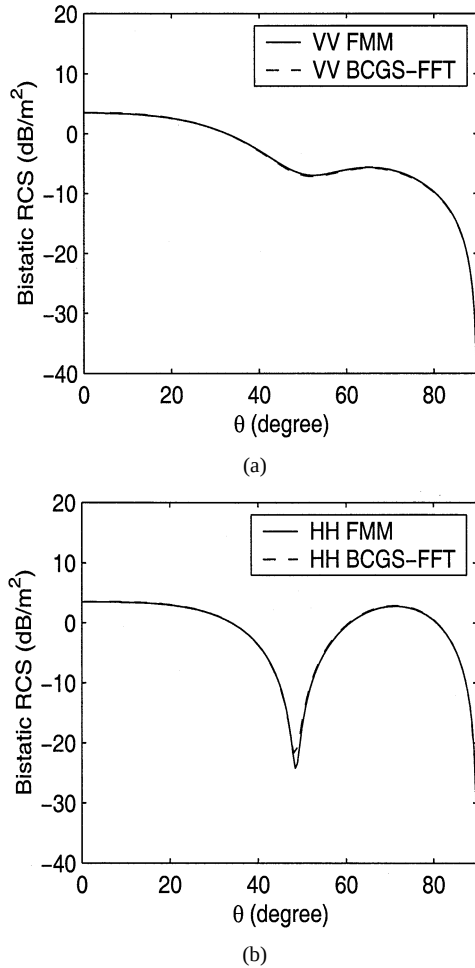


Fig. 3. Bistatic RCS from the sphere in Fig. 2. (a) $\phi_{sct} = 0$, VV polarization and (b) $\phi_{sct} = 0$, HH polarization.

points has to be taken for an accurate integration in the k_ρ domain. This problem can be attributed to the singularity in the Green's function. To accelerate the evaluation of the Green's function, the singularity term in the dyadic Green's function is extracted from its spectral domain expressions. This singularity term is exactly the direct term in the Green's function (the

same as the Green's function for a homogeneous background), and has a closed-form expression in the spatial domain. With this singularity subtraction, the length of the integration path for Sommerfeld integral evaluation to obtain the Green's function in spatial domain is typically reduced by a factor of 30 for the same accuracy. The singularity in the extracted direct term is removed by using spheric mean at the singular point, as in [5], [8], [9].

Interpolation: Secondly, due to the axial symmetry of the dyadic Green's function, we can reduce the evaluation of Green's function from a volume to a plane. Specifically, if the volume of the scattering object is of dimension $a \times b \times c$, in a straightforward evaluation the computational domain of the Green's function will be of dimension $2a \times 2b \times 2c$ (the factor 2 is necessary to avoid aliasing when using the FFT for convolution and correlation). However, noting that the Green's function is axially symmetric, one only needs to evaluate the Green's function over a vertical plane (i.e., the ρz plane) of size $\max(\sqrt{2}a, \sqrt{2}b) \times c$; the Green's function within the desired volume can then be obtained by a simple linear interpolation. This reduces the computational time from $O(N)$ to $O(N^{2/3})$, because the size of the computational domain is proportional to the number of unknowns (although for large-scale problem this CPU time can be negligible, for a small-scale problem this can have a substantial contribution because of the large proportion coefficient due to Sommerfeld integral).

E. Weak-Form Discretization

When solving the integral equation in (6), it is necessary to consider the second-derivative operator ($\nabla \nabla \cdot$) in (7). This operator increases the singularity of the Green's function and may pose a problem for the convergence of the iterative solution.

To overcome this problem, we adopt the weak-form discretization [5] where both the unknown electric flux density in the EFIE and the magnetic vector potential are expanded in terms of basis functions. This approach is similar to the problem of scattering from objects in the free space [5], [8]–[10]. In this work, both basis and testing functions are “roof-top” functions $\{\mathbf{b}_n(\mathbf{r})\}$ [18]. The detailed process for the discretization is the same as the “weak-form” discretization for free-space models [5], [8]–[10] except that the dyadic Green's function for a planar layered medium is used. Specifically, we expand

$$\mathbf{D}(\mathbf{r}) = \sum_{n=1}^N d_n \mathbf{b}_n(\mathbf{r}), \quad \mathbf{A}(\mathbf{r}) = \sum_{n=1}^N a_n \mathbf{b}_n(\mathbf{r}) \quad (14)$$

and then test (6) and (3) using testing functions $\{\mathbf{b}_m(\mathbf{r})\}$. Using (3), the discretized (6) becomes

$$\begin{aligned} e_{m,n,p}^{\text{inc},x} = & \sum_{i=1}^3 \left[b_i^x d_{m+i-2,n,p}^x + c_i^x \tilde{A}_{m+i-2,n,p}^x \right] \\ & + \sum_{i=1}^2 \sum_{j=1}^2 t_{i,j}^z \tilde{A}_{m+i-2,n+j-1,p}^y \\ & + \sum_{i=1}^2 \sum_{k=1}^2 t_{i,j}^y \tilde{A}_{m+i-2,n,p+k-1}^z \end{aligned} \quad (15)$$

$$\begin{aligned}
e_{m,n,p}^{\text{inc},y} = & \sum_{i=1}^2 \sum_{j=1}^2 t_{i,j}^z \tilde{A}_{m+i-1,n+j-2,p}^x \\
& + \sum_{j=1}^3 \left[b_j^y d_{m,n+j-2,p}^y + c_j^y \tilde{A}_{m,n+j-2,p}^y \right] \\
& + \sum_{i=1}^2 \sum_{k=1}^2 t_{i,j}^x \tilde{A}_{m,n+j-2,p+k-1}^z \quad (16)
\end{aligned}$$

$$\begin{aligned}
e_{m,n,p}^{\text{inc},z} = & \sum_{i=1}^2 \sum_{k=1}^2 t_{k,i}^y \tilde{A}_{m+i-1,n,p+k-1}^x \\
& + \sum_{j=1}^2 \sum_{k=1}^2 t_{k,j}^x \tilde{A}_{m,n+j-1,p+k-2}^y \\
& + \sum_{k=1}^3 \left[b_k^z d_{m,n,p+k-2}^z + c_k^z \tilde{A}_{m,n,p+k-2}^z \right] \quad (17)
\end{aligned}$$

where the coefficient vectors $\mathbf{b}^q$ and $\mathbf{c}^q$ and the coefficient matrix $\mathbf{t}^q$ are given by

$$\begin{aligned}
\mathbf{b}^q &= \frac{\Delta x \Delta y \Delta z \tilde{\epsilon}_i}{6} \begin{pmatrix} \tilde{\epsilon}_{m-\delta_{xq},n-\delta_{yq},p-\delta_{zq}}^{-1} \\ 2 \left(\tilde{\epsilon}_{m-\delta_{xq},n-\delta_{yq},p-\delta_{zq}}^{-1} + \tilde{\epsilon}_{m,n,p}^{-1} \right) \\ \tilde{\epsilon}_{m,n,p}^{-1} \end{pmatrix} \\
\mathbf{c}^q &= \Delta x \Delta y \Delta z \left[\frac{k_i^2}{6} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + (\Delta q)^{-2} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \right] \\
\mathbf{t}^q &= \Delta q \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}
\end{aligned}$$

with δ_{xq} , δ_{yq} and δ_{zq} being the regular Kronecker Delta function. In the above, the values of $e_{m,n,p}^{\text{inc},q}$ are related to $E_{m,n,p}^{\text{inc},q}$ ($q = x, y, z$) as

$$\begin{aligned}
e_{m,n,p}^{\text{inc},q} = & \frac{\Delta x \Delta y \Delta z}{6} \left[E_{m+\delta_{xq},n+\delta_{yq},p+\delta_{zq}}^{\text{inc},q} + 4E_{m,n,p}^{\text{inc},q} \right. \\
& \left. + E_{m-\delta_{xq},n-\delta_{yq},p-\delta_{zq}}^{\text{inc},q} \right] \quad (18)
\end{aligned}$$

and the quantities $d_{m,n,p}^q$, $\tilde{A}_{m,n,p}^q$ and $E_{m,n,p}^{\text{inc},q}$ ($q = x, y, z$) are

$$d_{m,n,p}^q = \frac{1}{\tilde{\epsilon}_i} D_p \left(\mathbf{r}_{m,n,p} - \frac{\hat{q} \Delta q}{2} \right) \quad (19)$$

$$\tilde{A}_{m,n,p}^q = \frac{1}{j\omega\mu_i\tilde{\epsilon}_i} A_q \left(\mathbf{r}_{m,n,p} - \frac{\hat{q} \Delta q}{2} \right) \quad (20)$$

$$E_{m,n,p}^{\text{inc},q} = E_q^{\text{inc}} \left(\mathbf{r}_{m,n,p} - \frac{\hat{q} \Delta q}{2} \right) \quad (21)$$

where $\hat{q}$ is the unit vector for the q (x, y , or z) direction.

The resulting discrete linear system can be written compactly as

$$\mathcal{L}^d[\mathbf{d}] = \mathbf{e}^{\text{inc}} \quad (22)$$

where $\mathbf{d}_i$ and $\mathbf{e}_i^{\text{inc}}$ are vectors of the unknown electric flux density and background electric field, respectively, and $\mathcal{L}^d[\cdot]$ is the discrete version of the linear operator $\mathcal{L}[\cdot]$ in (7) and is defined by (15)–(17) along with the discretized (3).

This process avoids taking any derivative of the Green's function in the EFIE and results in a less singular integral kernel. [The only integration with the Green's function is to obtain A_q in (3).] This reduced singularity improves the convergence rate when an iterative method is used to solve the discrete linear system.

F. Iterative Solver for the Discrete System

The linear (22) is then solved efficiently by the stabilized biconjugate gradient (BiCGSTAB) procedure [19]. As investigated by Xu *et al.* [10], for the volume integral equation, there is a strong evidence that the BiCGSTAB method converges much faster than CG [20], BiCG [21], conjugate gradient squared (CGS) [22], and the transpose-free quasiminimum residual (TFQMR) methods [23].

After discretization, the convolution and correlation kernels in the EFIE are transformed into discrete convolution and correlation kernels. Using the convolution and correlation theorems, these discrete convolution and correlation can be achieved efficiently by using the FFT algorithm. Therefore, during each iteration of the Bi-CGSTAB procedure, the computational cost for the operation $\mathcal{L}^d[\mathbf{d}_i]$ is only $O(N \log N)$. Consequently, the total computational cost of this BCGS-FFT method is $O(KN \log N)$, where K is the number of iterations. Furthermore, there is no need to store an MoM matrix; only vector $\mathbf{d}_i$ and a few auxiliary vectors of dimension N are required, leading to a memory requirement of $O(N)$ in the BCGS-FFT algorithm. Moreover, note that the number of layers only affects the computation of the Green's function, which only needs to be obtained once and is a small fraction of the total computational cost. Therefore, the computer memory requirement does not change, while the CPU time increases little with the increase of the number of layers.

III. NUMERICAL RESULTS

In this work, we focus on the applications in far-field measurements, i.e., when the source and observer are far from the targets embedded in the layered medium. Therefore, the excitation source is considered as a plane wave incident from the top layer. This scenario is highly relevant in the detection of buried objects and targets under trees.

A. Validation

Validation has been done in [12] for electromagnetic scattering from cuboids buried in two- and three-layer lossy media with the background field generated by a z directed electric dipole by comparing with independent FDTD results [24]. Here we show a further validation against the fast multipole method when the source is a plane wave.

Fig. 2 shows the geometry of a homogeneous sphere located in air with the ground surface at $z_1 = 0$ m; the soil has $\epsilon_{r2} = 5.0 - 0.2j$, $\mu_{r2} = 1.0$, $\sigma_2 = 0$ S/m. The sphere ($\epsilon_r = 3.0$, $\sigma = 0.001$ S/m) has a radius of $r = 0.3$ m and is centered at $(0, 0, 0.7)$ m. A 300 MHz plane wave is incident at $\theta_i = 0$, $\phi_i = 0$. To validate the BCGS-FFT method, the RCS results are compared with those obtained by the multilevel fast multipole algorithm in [25]. Fig. 3(a) and (b) show the RCS of VV and

HH polarizations, respectively. The results from the BCGS-FFT method have excellent agreement with those of the independent FMM code.

B. Cylinder in the Top Layer of a Lossy Half Space

Fig. 4 shows a circular cylinder in the top layer of a lossy half space characterized by $\epsilon_{r1} = 1.0$, $\mu_{r1} = 1.0$, $\sigma_1 = 0$ S/m, $z_1 = 0$ m; $\epsilon_{r2} = 5 - 0.2j$, $\mu_{r2} = 1.0$, $\sigma_2 = 0$ S/m. The radius and height of the cylinder are $r = 0.5$ m and $h = 4$ m, respectively. The electrical parameters of the cylinder are $\epsilon_r = 3$, $\sigma = 0.001$ S/m. The distance from the bottom of the cylinder to the interface of the half space is $d = 0.5$ m. The incident angle of the 519.6 MHz plane wave is $\theta_i = 45^\circ$ and $\phi_i = 0$.

This problem is discretized with $N = 1,285,920$ unknowns. Note that for simplicity, the number of unknowns in this paper is counted as the total number of scalar field variables inside the cuboid containing the target; thus, this includes the unnecessary unknowns in the small volume between the circular cylinder and the cuboid discretization volume. Fig. 5(a) shows the RCS versus the azimuth angle ϕ_{sct} when $\theta_{\text{sct}} = 45^\circ$ for VV and VH polarizations. Fig. 5(b) shows the convergences curve of the BCGS-FFT method. The code takes 350 and 367 iterations for the relative residual error to become less than 0.1% for TE and TM polarizations, respectively.

C. Computational Complexity

Fig. 6(a) and (b) give the log-log plot of the RAM requirements and CPU time per iteration as a function of N . The sampling rate is kept constant at 16 points per wavelength, the frequency changes from 173.2 to 692.8 MHz and the electrical size of the target varies from $1\lambda_g \times 1\lambda_g \times 4\lambda_g$ to $4\lambda_g \times 4\lambda_g \times 16\lambda_g$, where λ_g is the dielectric wavelength (the dielectric constant is 3) inside the cylinder. The number of unknowns for the largest model is $N = 2 \times 64 \times 63 \times 255 + 63 \times 63 \times 256 = 3,072,384$ when $f = 692.8$ MHz. The measurements of the CPU time and the RAM requirements are done on a Sun Ultra 60 workstation with 1.2 GBytes memory. Note that the plots in Fig. 6 confirm that the RAM and CPU requirements for the BCGS-FFT method are $O(N)$ and $O(N \log N)$, respectively.

D. Bunker Buried in a Half Space and in a Three-Layer Medium

Fig. 7 shows a bunker in three-layer medium characterized by $\epsilon_{r1} = 1.0$, $\mu_{r1} = 1.0$, $\sigma_1 = 0$ S/m; $\epsilon_{rm} = 5.0 - 0.2j$, $\mu_{rm} = 1.0$, $\sigma_m = 0$ S/m, $z_2 = 0$ m; $\epsilon_{r2} = 2.0$, $\mu_{r2} = 1.0$, $\sigma_2 = 0$ S/m. Note that this configuration can be used to simulate two scenarios: (a) for the three-layer medium, we set $z_1 = 3$ m; (b) for a two-layer half space, we set $z_1 = z_2 = 0$ so that the middle layer (index m) is absent. The dimension of the outside wall of the bunker is $21 \text{ m} \times 21 \text{ m} \times 9 \text{ m}$. The dimension of the room is $19 \text{ m} \times 19 \text{ m} \times 7 \text{ m}$. The electrical parameters of the wall are $\epsilon_r = 3$, $\mu_r = 1.0$, $\sigma = 0.05$ S/m. The room is filled with air. The distance from the top of the bunker to the interface at z_2 is $d = 2.5$ m. The 100 MHz plane wave is incident at $\theta_i = 0$ and $\phi_i = 0$. The electrical size of this problem is $7\lambda \times 7\lambda \times 3\lambda$, where λ is the wavelength of the incident electric field in free space.

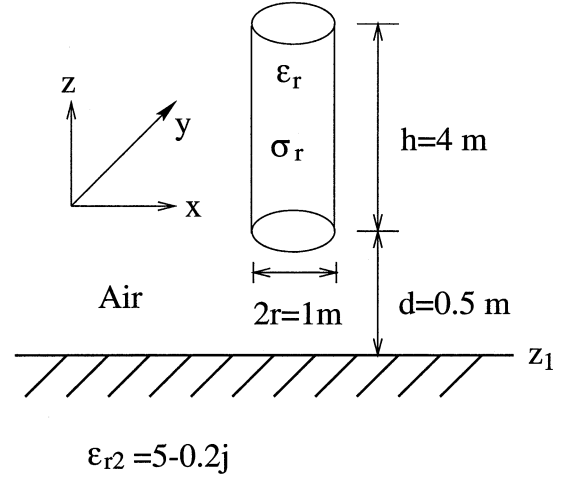


Fig. 4. Configuration of a finite cylinder in a lossy half space. The cylinder has $\epsilon_r = 3$, $\sigma = 0.001$ S/m.

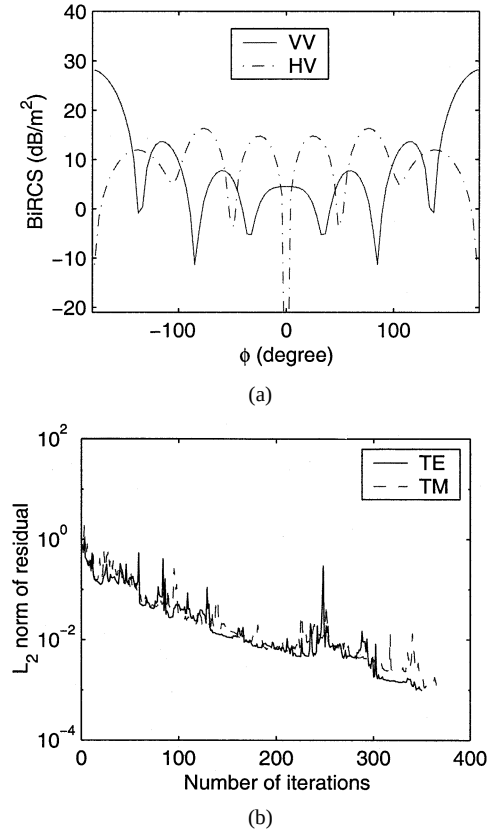


Fig. 5. Results from the BCGS-FFT method for Fig. 4. (a) Bistatic RCS when $\theta_{\text{sct}} = 45^\circ$ and (b) the convergence curves.

The number of unknown is $N = 2 \times 81 \times 80 \times 53 + 80 \times 80 \times 54 = 1,032,480$, which gives the sampling rates of 11.6 points/ λ in the x and y directions and 18 points/ λ in the z direction. Fig. 8(a) shows the bistatic RCS as a function of θ_{sct} when $\phi_{\text{sct}} = 0$ for VV polarization with the two-layer and three-layer background media, respectively. The horizontal (i.e., x or y direction) size of the bunker is roughly 7 wavelengths in the air, which corresponds to the seven peaks in (8a). Fig. 8(b) is the convergence curve of the BCGS-FFT method for these two scenarios. The code takes 141 and 125 iterations, respectively,

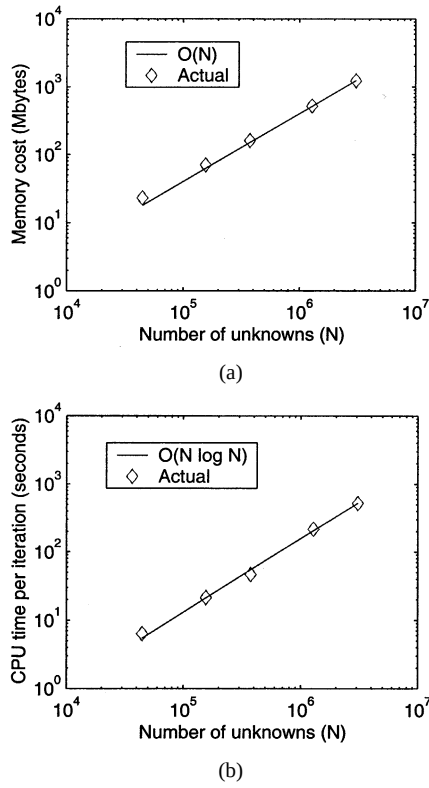


Fig. 6. Computer memory (a) and CPU time per iteration and (b) versus the number of unknowns for the case in Fig. 4.

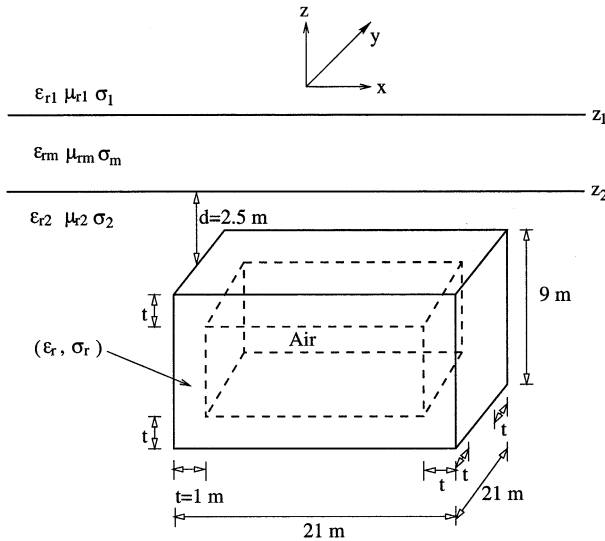


Fig. 7. Geometry of a bunker in a two- or three-layer background characterized by $\epsilon_{r1} = 1.0$, $\mu_{r1} = 1.0$, $\sigma_1 = 0$ S/m; $\epsilon_{rm} = 5.0 - 0.2j$, $\mu_{rm} = 1.0$, $\sigma_m = 0$ S/m, $z_2 = 0$ m; $\epsilon_{r2} = 2.0$, $\mu_{r2} = 1.0$, $\sigma_2 = 0$ S/m. For the two-layer case, $z_1 = z_2 = 0$; for the three-layer case, $z_1 = 3$ m.

for the L_2 norm of the relative residual error to become less than 0.1%. The run time is 5.1 and 4.43 CPU hours, respectively, on an SGI Origin 2400 processor.

E. Modeling of Buried UXO Objects

Fig. 9 shows a simple model of an UXO cylindrical object buried in a three-layer background characterized by $\epsilon_{r1} = 1.0$,

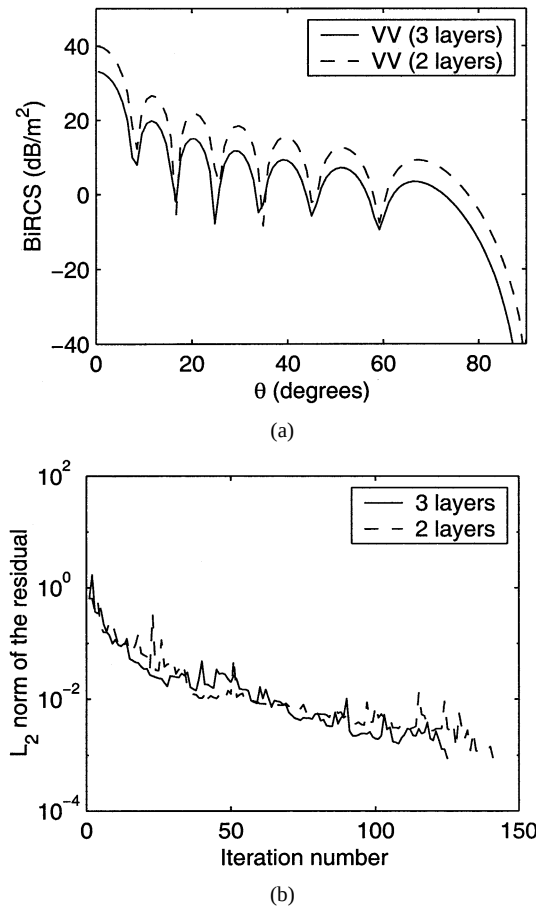


Fig. 8. Results from the BCGS-FFT method for a bunker in a two- or three-layer background shown in Fig. 7. (a) Bistatic RCS when $\phi_{sct} = 0$ and (b) the convergence curves.

$\mu_{r1} = 1.0$, $\sigma_1 = 0$ S/m, $z_1 = 2$ m; $\epsilon_{r2} = 2.56$, $\mu_{r2} = 1.0$, $\sigma_2 = 0.04$ S/m, $z_2 = 0$ m; $\epsilon_{r3} = 5 - 0.2j$, $\mu_{r3} = 1.0$, $\sigma_3 = 0$ S/m. The radius and height of the cylinder are $r = 0.5$ m and $h = 4$ m, respectively. The electrical parameters of the cylinder are $\epsilon_r = 3$, $\sigma = 0.01$ S/m. The distance from the top of the cylinder to the interface of the layer 2 and 3 is $d = 0.5$ m. The incident angle of the 519.6 MHz plane wave is $\theta_i = 45^\circ$ and $\phi_i = 0$.

The problem is discretized with $N = 1,285,920$ unknowns. Fig. 10 shows the bistatic RCS as a function of azimuth angle ϕ_{sct} when $\theta_{sct} = 45^\circ$ for VV and VH polarization. The code takes 84 iterations for the relative residual error to become less than 0.1% for an incident plane wave of TM polarization. The contrast of the background is relatively large and this results in strong reflection of the incident and scattered field at the interfaces z_1 and z_2 . Therefore, the bistatic RCS is low as shown in 10. This low RCS indicates that the buried UXO is difficult to detect under this condition. Fig. 11(a) shows the RCS results with a reduced background contrast with $\epsilon_{r2} = 1.44$, $\sigma_2 = 0$ S/m; $\epsilon_{r3} = 1.69$, $\sigma_3 = 0$ S/m (lossless). The other parameters are the same as that in Fig. 9. Note that the RCS is much higher with this lossless background of reduced contrast because the reflections at the interfaces and the loss due the conductivity of the background are reduced. This increased RCS (i.e. scattered field) makes it is easier to detect the buried UXO. The convergence curves for Fig. 10 and Fig. 11(a) are

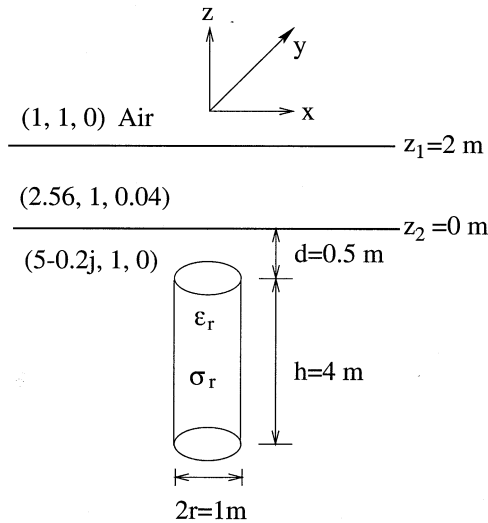


Fig. 9. Configuration of a finite cylinder ($\epsilon_r = 3$, $\sigma = 0.01$ S/m) buried in a three-layer background.

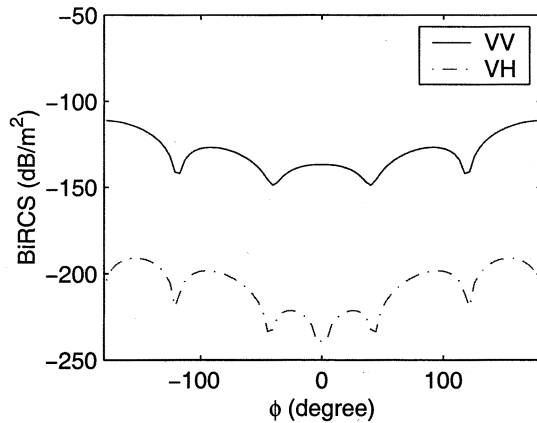
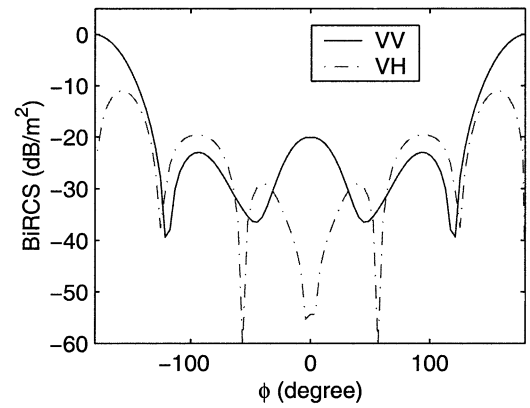


Fig. 10. RCS from the BCGS-FFT method for Fig. 9 with $\theta_{\text{sct}} = 45^\circ$.

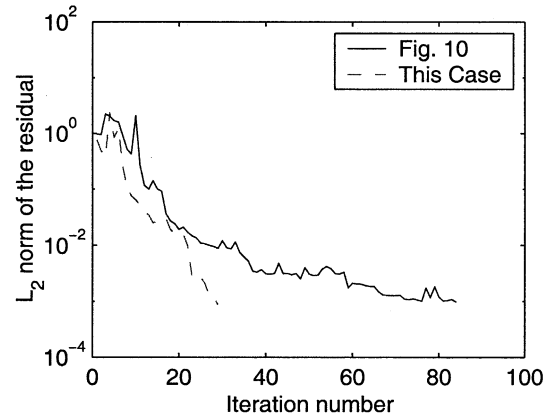
shown in Fig. 11(b). As shown in this study, this fast solver provides a useful tool for the users to determine the detectability of objects buried in the ground or obscured by trees.

F. Discussions

The fast three dimensional (3-D) volume integral equation solver presented here uses the important splitting of Green's function into convolutional and correlational terms to allow the use of FFT algorithm for fast matrix-vector multiplication. At present, this splitting is limited to the case where the inhomogeneous scatterer is completely embedded within one layer. Although this already includes many applications, the more general cases where the scatterer traverses across two or more layers cannot be directly accelerated by the current version of the fast solver. One straightforward remedy to this is summarized below in a new iteration loop: 1) First, the scatterer is divided into several subscatterers, each of which is embedded in only one layer. b) Then, the induced current on each subscatterer is solved using the above method by assuming the primary source and the current induced in other subscatterers from the previous iteration as the "incident" field. After all induced currents in all subscatterers are solved, we obtain the current in the primary scatterer



(a)



(b)

Fig. 11. Results from the BCGS-FFT method. The configuration is the same as in Fig. 9 except that the background contrast is reduced to $\epsilon_{r2} = 1.44$, $\sigma_2 = 0$ S/m, $\epsilon_{r3} = 1.69$, $\sigma_3 = 0$ S/m. (a) RCS for $\theta_{\text{sct}} = 45^\circ$ and (b) the convergence curves for Fig. 10 and for this case.

within the present iteration. 3) These induced currents, together with the primary source, are then used as the source for the next iteration. This process continues until the induced current converges. Such an iterative method as well as other methods will be investigated in the future for more general cases.

IV. CONCLUSION

The main contributions of this work include the development of a fast 3-D solver for inhomogeneous objects embedded in multilayer media and the applications in far-field simulations of such objects. We incorporate the stabilized biconjugate gradient method in the FFT accelerated volume integral equation solver. The BCGS-FFT method is applied to solve electromagnetic subsurface sensing of inhomogeneous objects embedded in a general planarly layered medium. The CPU time of this method is $O(N \log N)$ and the memory cost is $O(N)$. Large-scale problems of over 3 million unknowns have been solved on a SUN Ultra 60 workstation. This method is an appealing choice for large-scale scattering problems in layered media.

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