

RAPID SOLUTION OF HYBRID SURFACE-VOLUME INTEGRAL EQUATIONS FOR EM SCATTERING BY MULTILEVEL FAST MULTIPOLE ALGORITHM

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1. Introduction

Numerical modeling of electromagnetic scattering from complex targets is a cost-effective method for understanding the scattering characteristics of various radar targets. Recent advances in fast algorithms for computational electromagnetics demonstrated great potential to handle targets that are orders or magnitude larger in electrical size than those that can possibly be handled by conventional solvers using the same amount of computer resources. This is achieved by reducing the solvers' computational complexity--a function that measures the scaling properties of the CPU time and memory consumption in terms of the number of unknowns in a problem. However, significant reduction of computational complexity has been achieved for scattering problems with conducting targets only. To expand the capability of fast solvers to include material or material coated targets, this paper applies the multilevel fast multipole algorithm to accelerate the solution of the hybrid integral equations. It is known that the basic idea in the fast multipole method is in the expansion of the free-space Green's function in terms of multipoles. As a result, to extend the fast multipole method for solving problems with materials, it is easier, for book-keeping purpose, to preserve the free-space Green's function in the formulation of the integral equations. A natural way to realize this is to use the volume integral equation for the material region.

In the following, we present we will present the formulations and implementations that are specific to the hybrid integral equations since the basic concept of the fast multipole method (FMM) and its multilevel implementation have been reported elsewhere [5-8]. The discussion is in the frequency domain, and the time factor of $\exp\{-i\omega t\}$ has been suppressed from the equations.

2. General Formulation

In integral equation approaches, the fields are calculated from current distributions through spatial integration. For the model introduced in this paper, there are two types of currents. One is a surface current $\bar{J}_s$ that is induced on the conducting surfaces. The other is the polarization (or volume) current $\bar{J}_v$ induced in the dielectric. The current distributions depend on several factors such as the geometry of the structure, the frequency of the sources, the material characteristics of the dielectric, etc. They are determined by the following two integral equations [1,2],

$$\hat{n} \times \left[\bar{\bar{L}}_s(\bar{r}, \bar{r}') \cdot \bar{J}_s(\bar{r}') + \bar{\bar{L}}_v(\bar{r}, \bar{r}') \cdot \bar{J}_v(\bar{r}') \right] = -\hat{n} \times \bar{E}^{\text{inc}}(\bar{r}), \quad \bar{r} \in S, \quad (1)$$

$$\bar{\bar{L}}_s(\bar{r}, \bar{r}') \cdot \bar{J}_s(\bar{r}') + \bar{\bar{L}}_v(\bar{r}, \bar{r}') \cdot \bar{J}_v(\bar{r}') - \bar{E}(\bar{r}) = -\bar{E}^{\text{inc}}(\bar{r}), \quad \bar{r} \in V. \quad (2)$$

In the above: $\hat{n}$ is a unit vector on the conducting surface S ; $\bar{E}^{\text{inc}}$ is the field generated by a transmitting radar; and $\bar{\bar{L}}_s$ and $\bar{\bar{L}}_v$ are defined by

$$\bar{\bar{L}}_\alpha(\bar{r}, \bar{r}') \cdot \bar{J}_\alpha(\bar{r}') = \int_\alpha \bar{\bar{G}}(\bar{r}, \bar{r}') \cdot \bar{J}_\alpha(\bar{r}') d\bar{r}', \quad \bar{r} \in \alpha, \quad \alpha = S \text{ or } V, \quad (3)$$

where the dyadic Green's function $\bar{\bar{G}}(\bar{r}, \bar{r}') = i\omega\mu_b(\bar{I} + \nabla\nabla/k_b^2)g(\bar{r}, \bar{r}')$, with $g(\bar{r}, \bar{r}') = \exp\{ik_b|\bar{r} - \bar{r}'| \}/4\pi|\bar{r} - \bar{r}'|$, and k_b being the wavenumber of the background material. It should be noted that the total electric field $\bar{E}(\bar{r})$ on the left-hand side of equation (2) is related to the volume current by $\bar{J}_v(\bar{r}) = i\omega(\epsilon_0 - \epsilon) \cdot \bar{E}(\bar{r})$ with ϵ being the permittivity of the coating dielectric. The integral equations in (1) and (2) are called hybrid surface-volume integral equations.

The method of moments is then applied to discretize the hybrid integral equation into a matrix system. To this end, we select to use flat triangular patches for the surface mesh, and the tetrahedron to represent the dielectric volume region. This choice has two advantages: (1) The mesh can accurately represent any complex objects, (2) it is easy to realize the connection condition that is required to solve the hybrid integral equations. As a result, the Rao-Wilton-Glisson [3,4] basis functions are used to expand the currents, as given by

$$\bar{J}_s(\bar{r}) = \sum_{n=1}^{N_s} a_n^s \bar{f}_n^s(\bar{r}), \quad \bar{J}_v(\bar{r}) = \sum_{n=N_s+1}^{N_s+N_v} a_n^v \chi(\bar{r}) \bar{f}_n^v(\bar{r}),$$

where the $\bar{f}$'s are the basis functions, and the a 's are the expansion coefficients (unknown constants). The above expressions are then substituted into the coupled integral equation (1) and (2). Testing the resultant equations gives rise to a matrix equation that can be formally written as $\sum_{j=1}^N A_{ij} a_j = b_i$. Here x_j is the expansion coefficient, A_{ij} and b_j are given by

$$A_{ij} = \int_{V_i} \bar{t}_i(\bar{r}) \cdot \int_{V_j} \bar{\bar{G}}(\bar{r}, \bar{r}') \cdot \chi \bar{f}_j(\bar{r}') d\bar{r}' d\bar{r}, \quad b_i = -\int_{V_i} \bar{t}_i(\bar{r}) \cdot \bar{E}^{\text{inc}}(\bar{r}) d\bar{r}.$$

It should be noted that in (4) and (5), the basis function, $\bar{f}_j$, and the testing function, $\bar{t}_i$, are defined either on surface or volume cells, depending on the shapes of V_j and V_i , respectively. However, if V_j is surface cell, $\chi(\bar{r}) = 1$, otherwise, $\chi(\bar{r}) = (\epsilon_b - \epsilon)/\epsilon$. In the implementation of this paper, $\chi(\bar{r})$ is treated as constant across the small volume cell. The error caused by this approximation is small if the cell size is much less than a wavelength in that material. The

matrix equation, $\sum_{j=1}^N A_{ij} a_j = b_i$, can be solved either directly using a matrix inversion algorithm or iteratively using an iteration solver such as the conjugate gradient (CG) method. In this paper we apply the fast multipole method (FMM) to accelerate the solution process. The basic idea of FMM is to spatially group the basis functions, and transfer the basis-to-basis interaction mechanism into group-to-group mechanism for well-separated groups. In what follows it is assumed that the cells (both surface and volume) are grouped into M groups, the testing function $\bar{t}_i$ belong to group m , and the basis function $\bar{f}_j$ belong to group n . If the group distance r_{mn} is small, the matrix elements A_{ij} 's are called near-neighbor elements. Otherwise they are called non-near-neighbor elements. For near-neighbor elements, A_{ij} can be expressed in mixed potential representation,

$$A_{ij} = i\omega\mu_b \int_{V_i} \bar{t}_i(\bar{r}) \cdot \left[\int_{V_j} g(\bar{r}, \bar{r}') \chi \bar{f}_j d\bar{r}' + \frac{\nabla}{k_b^2} \int_{V_j} g(\bar{r}, \bar{r}') \nabla' \cdot (\chi \bar{f}_j) d\bar{r}' \right] d\bar{r}. \quad (4)$$

The non-near-neighbor element can be written as [8]

$$A_{ij} = \eta_b \left(\frac{ik_b}{4\pi} \right)^2 \int d^2 \hat{k} \bar{V}_{fmi}(\hat{k}) \cdot T_{mn}(\hat{k}) \cdot \bar{V}_{snj}(\hat{k}) \quad (5)$$

where the integration over $\hat{k}$ is on a unit sphere, and

$$\bar{V}_{fmi}(\hat{k}) = \int_{V_i} \left(\bar{I} - \hat{k}\hat{k} \right) \cdot \bar{t}_i(\bar{r}) e^{i\bar{k} \cdot (\bar{r} - \bar{r}_m)} d\bar{r} \quad (6)$$

$$T_{mn}(\hat{k}) = \sum_{l=0}^L i^l (2l+1) h_l^{(1)}(kr_{mn}) P_l(\hat{k} \cdot \hat{r}_{mn}) \quad (7)$$

$$\bar{V}_{snj}(\hat{k}) = \int_{V_j} \left(\bar{I} - \hat{k}\hat{k} \right) \cdot \bar{f}_j(\bar{r}') \chi(\bar{r}') e^{-i\bar{k} \cdot (\bar{r}' - \bar{r}_n)} d\bar{r}' \quad (8)$$

In the above, $\bar{r}_m$ and $\bar{r}_n$ are the center position of group m and group n , respectively, $\bar{r}_{mn} = \bar{r}_m - \bar{r}_n$, and the selection of L can be found in [6]. Note that because the appearance of the factor χ in (8), the symmetry between $\bar{V}_{fmi}$ and $\bar{V}_{sni}$ no longer exists.

3. Numerical Results and Summary

A numerical example is presented in this section to demonstrate the implementation of the multilevel fast multipole algorithm (MLFMA) for solving the hybrid integral equation of electromagnetic scattering. As shown in Figure 1, the RCS calculated using the MLFMA is compared with the exact solution for a conducting sphere coated with dielectric material. The Conjugate Gradient (CG) method is used to solve the matrix equation iteratively. The memory requirement for this problem is 218 MB by a two-level MLFMA, and the CPU time per iteration is 6.97 sec. on a SUN Ultra5 workstation. This problem cannot be solved using the same computer if a direct method is used because the number of unknowns to discretize the problem is $N=19992$. The memory needed to store this a complex matrix of this size is about 2.8GB.

To summarize, the multilevel fast multipole algorithm has been applied for solving hybrid integral equations. Tetrahedron volume cells are used to discretize the material region, and the triangle patches are used to represent the conducting surfaces. This selection provides accurate

and flexible representation for any complex object shapes. Due to the reduction of the computational complexity, the method presented in this paper can be applied to solve electromagnetic scattering problems with electrically larger dielectrics.

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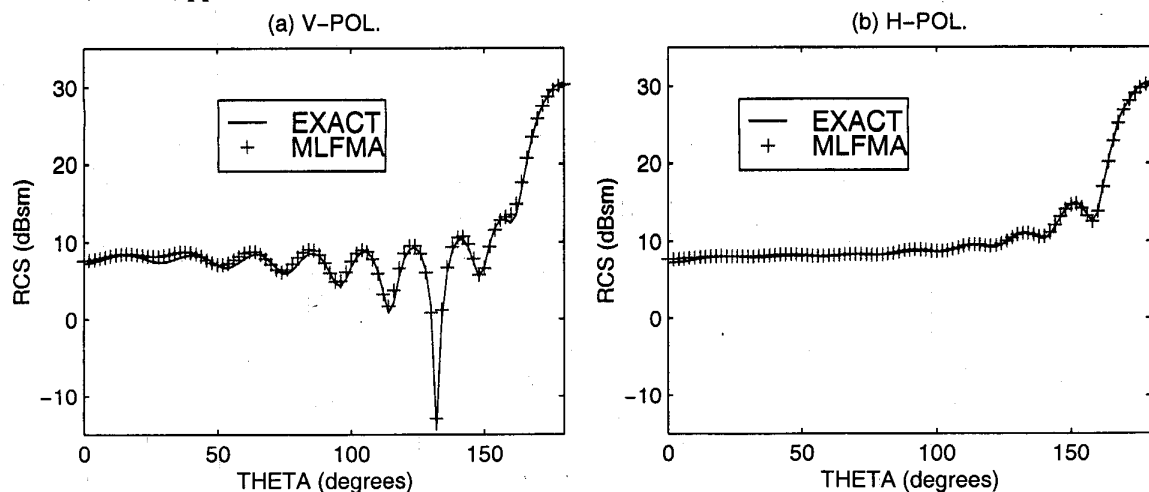


Figure 1. Comparison of the bi-static RCSs for a conducting sphere coated with dielectric material. The sphere radius is $1.5\lambda_0$ and the coating material thickness is $0.1\lambda_0$. The coating material has permittivity of $\epsilon_r = 1.5 + 0.5i$. The incident direction is $\theta = 0^\circ$. Incident wave frequency is $f = 300$ MHz. (a) Vertical polarization, (b) Horizontal polarization.