

Comparison of Different Implementations of VIE Method

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1. Introduction

Recently, a volume integral equation (VIE) formulation was developed, which can be used for the analysis of scattering from general material bodies [1]. Conductors are treated as a dielectric by letting the permittivity approach infinity. The formulation uses the dyadic Green's function, which is singular when the source and field points coincide. Removing this singularity gives a source dyadic term that is geometry dependent [2]. In application of the moment methods to solve the volume integral equation, using pulse type basis functions, one requires to sub-divide the object into small cells. With different cell shapes one needs to use the corresponding source dyadic terms, that may influence the performance of the solution algorithm. In this paper, we present analysis of dependence of the solution on the cell shape and size.

2. Different Implementations of VIE formulation

Assume a material object is illuminated by a plane wave E^{inc} . A solution for this problem can be obtained by using the equivalence theorem and the volume integral equation. In this method, the object is replaced by a volume polarization current J_p , which radiates in free space, and generates the scattered field. A free space dyadic Green's function is used and the scattered field can be expressed as [3]:

$$E^{sc} = \int_V (J_p(r') \cdot \Psi(r, r')) dV \quad (1)$$

where

$$\Psi(r, r') = -j\omega\mu_0 \left[\tilde{I} + \frac{\nabla\nabla}{k_0^2} \right] G(r, r') \quad (2)$$

and $k_0 = \omega(\mu_0\epsilon_0)^{\frac{1}{2}}$, $G(r, r')$ is the free space Green's function, ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively. V is the volume of the object. According to the definition of the polarization current, we have

$$J_p(r) = j\omega(\epsilon - \epsilon_0)E^i \quad r \in V \quad (3)$$

or

$$E^t = \frac{J_p(r)}{j\omega\epsilon_0(\epsilon_r - 1)} \quad r \in V \quad (4)$$

where ϵ_r is relative complex dielectric constant of the medium, E^t is the total field inside the object, and the current J_p is polarization current flowing within the object. Substituting (1) into (4) and employing the concepts of 'principle value' and the source dyadic term in source region, we obtain the following integro-differential equation:

$$E^{inc}(r) + \lim_{\delta \rightarrow 0} \int_{(V-V_\delta)} (J_p(r') \cdot \Psi(r, r')) dV + \frac{\tilde{L} \cdot J_p(r)}{j\omega\epsilon_0} = \frac{J_p(r)}{j\omega\epsilon_0(\epsilon_r - 1)} \quad r \in V \quad (5)$$

where $\frac{\tilde{L} \cdot J_p(r)}{j\omega\epsilon_0}$ is the source dyadic term used to compensate for the removal of the singularity[2]. The calculation of the elements of impedance matrix involves the following integral:

$$I_{mn}(r) = \int_V \left(J_p(r') \frac{\partial^2 G}{\partial x'_m \partial x'_n} \right) dV \quad m, n = 1, 2, 3 \quad (6)$$

If an infinitesimal volume V_δ is chosen to enclose the singular point and to be removed, then integral (6) can be expressed as a sum of two terms [4]:

$$I_{mn}(r) = \lim_{V_\delta \rightarrow 0} \int_{(V-V_\delta)} \left(J_p(r') \frac{\partial^2 G}{\partial x'_m \partial x'_n} \right) dV + C_{mn} \quad m, n = 1, 2, 3 \quad (7)$$

The values of C_{mn} depend on the shape of the cells, and can be obtained for several special cases [4]:

1. a cube with the origin at its centre and arbitrary axes:

$$C_{mn} = -\frac{1}{3} J_p \delta_{mn} \quad (8)$$

2. a sphere with arbitrary origin and axes:

$$C_{mn} = -\frac{1}{3} J_p \delta_{mn} \quad (9)$$

3. a circular cylinder with the origin at its centre, radius a , height $2b$, and its axis coincident with the x_3 axis

$$C_{11} = C_{22} = -\frac{b}{2\sqrt{a^2 + b^2}} J_p \quad (10a)$$

$$C_{33} = -\left(1 - \frac{b}{\sqrt{a^2 + b^2}} \right) J_p \quad (10b)$$

$$C_{mn} = 0 \quad \text{if } m \neq n \quad (10c)$$

Selecting an appropriate cell shape and using its corresponding source term from equations (8) to (10), equation (7) with (5) gives a matrix for J_p to be solved.

3. Results and Comparison

In this section, we examine and compare the results by different implementations of the VIE method. The first example is a dielectric cube of 0.2λ sidelength. The dielectric constant ϵ_r is (4.0,0.0). The calculated E-plane and H-plane far fields by using different shapes of cells, i.e., cubes, right circular cylinder and sphere are shown as curve A, B and C in Fig.1. All results are compared with ones reported in [5], shown as D with circles, and show excellent agreement. But one can see that the curve A gives the most accurate results and curve C is the worst of the three. This is understandable, because using cubes as cells will best fit the geometry of the object in this example.

The second example is a lossy rectangular block, whose side are 0.4λ , 0.3λ , and 0.2λ , respectively. And its permittivity is $\epsilon_r = (5.0, -0.02)$. Again, the calculated E-plane and H-plane far fields are shown in Fig.2 as curve A, B and C by using cubes, circular cylinder, and sphere, respectively. One can see that the results by using spheres as cells degenerate significantly.

The third example is a circular cylindrical dielectric resonator antenna with a ground plane reported in [6], which is a combination of a dielectric and conductor. The dielectric is circular cylinder. The height is 0.1022λ , and the radius is 0.1706λ . The radius of the ground plane is 1.297λ . The feed is modelled as an electric dipole at the edge of the dielectric cylinder. The calculated E-plane and H-plane far fields are shown in Fig.3. Both curves A and B show good agreement with the results in [6]. And curve B agrees better than A, especially for H-plane field. Using spheres as cells fails to give good results, which are shown as curve C in Fig3.

In conclusion, to get better results, one must carefully select the shape of the cells. Generally, cubes work better for object of cubical or rectangular cylinders. Circular cylinders work better for the object of circular cylinder. And the spheres do not generally work well for these geometries. Other shapes of cells and their source dyadic terms need further study so that the VIE method can be used more efficiently and more accurately for arbitrary geometries.

References:

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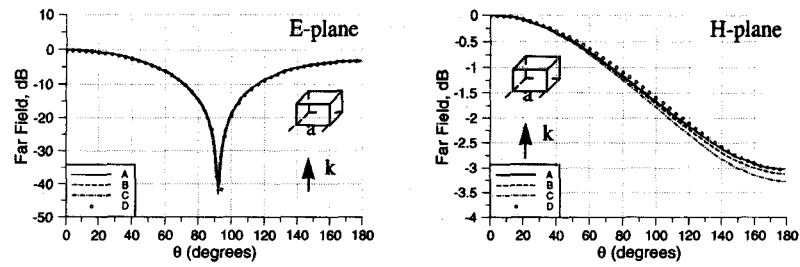


Fig.1 Far field of a dielectric cube of 0.2λ by using different implementations of VIE method

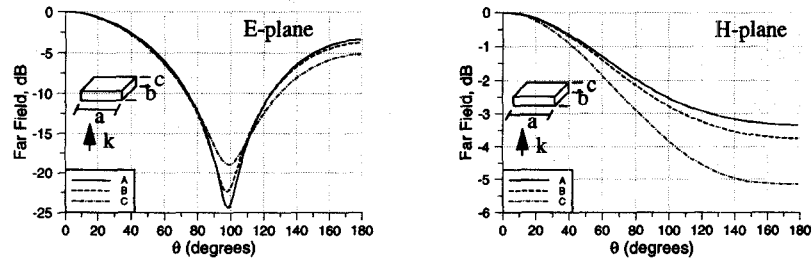


Fig.2 Far field of a rectangular dielectric block by using different implementations of VIE method

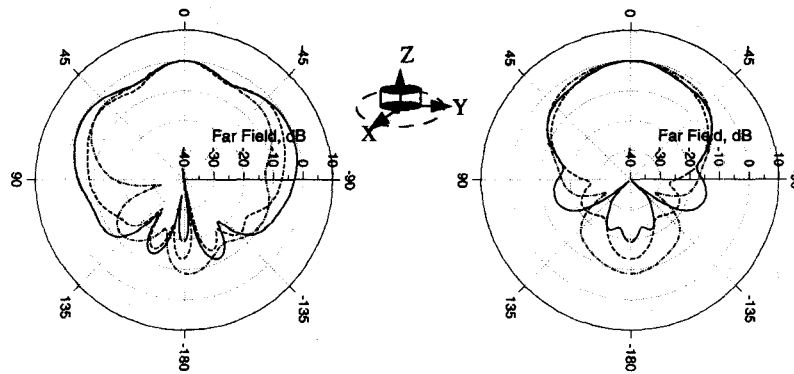


Fig.3 Far fields of a dielectric resonator antenna by different Implementations of VIE method
curve A-solid line, curve B-dashed line, curve C-dot-dashed line