

Dielectric Radome Analysis Using Multilevel Fast Multiple Algorithm¹

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I. Introduction

The presence of radome will generally affect the radiation characteristics of antennas. Techniques for analyzing the antenna pattern deformation by radomes are usually high frequency methods (such as GTD and PO). An important assumption of high frequency methods is that the structures have smooth surfaces and electrically large radius of curvature. For most portion of a realistic radome, this assumption is valid. However for some radome types that have sharp tips such as the ogive or cone, the radius of curvature may be comparable to or smaller than a wavelength, the high frequency approximation is not valid in that region. Numerical methods [1] such as the method of moments provide accurate solution to the problem compared to the high-frequency solution. However because the number of unknowns (N) increases rapidly with the electrical size of the radome structure, the numerical methods were either applied to two dimensional problems or applied to 3D problems with electrically small sized radomes. This is due to the high computational complexity of most direct numerical methods.

This paper presents a method that can be applied to efficiently and accurately analyze dielectric radomes of sizes much larger than that can be analyzed by a direct numerical method, and provides higher solution accuracy than a high frequency method in general. This method is based on the solution of the volume integral equation (VIE) by the multilevel fast multipole algorithm (MLFMA). It has the asymptotic computational complexity of $O(N \log N)$ when N is very large.

II. Algorithm Description

Consider a dielectric shell of arbitrary shape resides in a homogeneous background medium of infinite extent with ϵ_b, μ_b as permittivity and permeability parameters. The dielectric has position dependent permittivity $\epsilon(\mathbf{r})$. By equivalent principle, a volume integral equation is formulated as follows for the induced polarization current inside the dielectrics,

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + i\omega\mu_b \int_V G(\mathbf{r}, \mathbf{r}') \mathbf{J}_V(\mathbf{r}') dV' - \frac{\nabla}{i\omega\epsilon_b} \int_V G(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_V(\mathbf{r}') dV', \quad (1)$$

where V is the radome region, $\mathbf{E}^i$ is the antenna's primary radiation in the absence of the radome, $\mathbf{E}$ is the total electric field in space, $\mathbf{J}_V$ is the induced volume current in the radome

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given by $\mathbf{J}_v(\mathbf{r}) = -i\omega[\epsilon(\mathbf{r}) - \epsilon_b]\mathbf{E}(\mathbf{r})$, $G(\mathbf{r}, \mathbf{r}') = \exp\{ik_b R\}/(4\pi R)$ is the 3D scalar Green's function for the background medium, $R = |\mathbf{r} - \mathbf{r}'|$ is the distance from a source point $\mathbf{r}'$ to a field point $\mathbf{r}$, and $k_b = \omega\sqrt{\epsilon_b\mu_b}$ is the wavenumber of the background medium. In Equation (1), the two integrals on the right-hand side represents the radiation from the induced volume current and the corresponding charge. It is this radiation that affects the radiation pattern of the antenna. In other words, Equation (1) allows us to calculate the true radiation pattern for the antenna in the presence of radome if the induced current is determined.

The method of moments has been applied to solve the volume integral equation [6]. In applying a sub-domain method of moments, one needs to subdivide the dielectrics into small volume cells of a selected shape. Previous studies used cubic model [2], as well as tetrahedron models [3]. The conjugate gradient fast Fourier transform (CGFFT) [4] algorithm can be easily implemented using the structured cubic model. However it is not accurate in modeling structures with curved boundaries and the geometric modeling error results in large solution error if the dielectric contrast is high. On the other hand, the tetrahedron and hexahedron shapes are suitable for modeling arbitrarily shaped dielectric structures. Hence, In this paper both of the two shapes are considered in the implementation of the algorithm. If the matrix equation is solved by an iterative method such as the conjugate gradient (CG) method, the computational complexity for performing a matrix vector multiplication is $O(N^2)$ for an N -unknown problem. Apparently, because of this complexity, both memory requirement and CPU time can become prohibitively high for large N . Hence direct solution by an iterative solver for the set of linear equations is not appropriate for electrically large radomes. This is the most important reason that the MoM study of dielectric scattering problems are limited to small-sized structures (the conjugate gradient fast Fourier transform (CGFFT) algorithm can be used to reduced the matrix-vector multiplication operation to $O(N \log N)$. But this algorithm is efficient only when applied to dielectric objects of large fractional volume). On the other hand, the $O(N^2)$ complexity for a matrix vector multiplication can be reduced to $O(N \log N)$ for large N by the multilevel fast multipole algorithm (MLFMA) [5]. Since the integral equation (1) is formulated using the free-space Green's function, the implementation of the MLFMA for VIE is very similar to that for the surface integral equation for conducting objects [5,6]. The MLFMA is ideal for solving the scattering and radiation problems with dielectric objects that have small fractional volume such as the dielectric radomes.

III. Numerical Results

The MLFMA algorithm for solving the volume integral equation has been implemented for both tetrahedron and hexahedron meshes. In this section, some numerical examples are presented to demonstrate the application of the algorithm for analyzing the radiation patterns of dipoles in the presence of radomes. As a validation, the first example considers a spherical dielectric shell of uniform thickness for which analytical solution exists. The shell radius is 1.2 m and its thickness is 0.08 m. The dielectric permittivity of $2+i1$ is uniformly distributed in the shell. A 9-element uniform dipole array with quarter wavelength inter-element spacing is located on the z-axis inside the shell. The array operates at 300 MHz. The radiated field of the shell-array system is calculated for $\phi = 0^\circ$ and $\theta = 0^\circ$ to 90° at a distance of $r=100$ m. Figure 1 shows the comparison of the MLFMA results with the exact solution using Mie-series (the radiation of the array in free-

space is also plotted as reference). For radome shapes other than the spherical shell, there is no analytical solution. Hence the results from the surface integral equation are used as comparison. In figure 2, the radiated far field of a dipole in the presence of an ogive radome is plotted and compared with that from the surface integral equation. One can see the excellent agreement between the two solutions (as reference, the radiation of the dipole in free-space is also plotted in the figure).

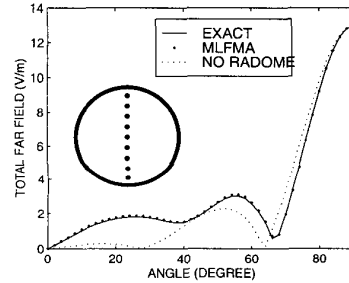


Fig. 1. The normalized radiation fields at $r=100$ m of a dipole array inside a spherical shell located on z-axis. The dipole element locations are shown as dots in the cross view of the spherical shell.

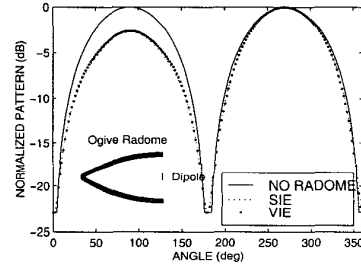


Fig. 2. The electric fields of a dipole in an ogive radome. The dipole is located at the bottom center of the radome and is oriented in the z-direction (perpendicular to the radome axis, as shown by a short line segment in figure).

In order to compare the interfering effects of different shapes of radomes to antenna radiation pattern, next example shows the calculated results for a dipole array in the presence of three types of radomes: (a) a tangent ogive with height $6.67\lambda_0$ and base diameter $5.33\lambda_0$ (modeled by 76,914 tetrahedrons), (b) a straight cone with the same dimension as the ogive (modeled by 55,308 tetrahedrons), and (c) a hemisphere of base diameter $5.33\lambda_0$ (modeled by 45,000 tetrahedrons). The thickness and the permittivity of all three radomes are the same. They are $t=0.008\lambda_0$, $\epsilon_r=2+li$. The dipole array is on z-axis and is made of three dipole elements of equal amplitude and phase. The locations of the elements are $-1.0\lambda_0, 0, 1.0\lambda_0$. The calculated results for $\phi=0$ and θ from 0 to 180° are shown in Figure 6. As is expected, the interferences by the ogive and cone shaped radomes are much severe compared to that by the hemisphere radomes.

VI. Summary

A fast algorithm based on the volume integral equation is described for the analysis of antenna-radome interaction problems. The algorithm is implemented for both tetrahedron and hexahedron meshes, and is applicable to model arbitrarily shaped 3D dielectric radomes. The application of the multilevel fast multipole algorithm (MLFMA) significantly reduced the memory and CPU

time requirement for solving the volume integral equation, and hence the algorithm presented in this paper can be applied to problems with electrically large sized radome. This algorithm is different from the ray tracing methods in that it yields highly accurate solution for radomes that contain small-scale geometric variations such as the sharp portion of the ogive radomes.

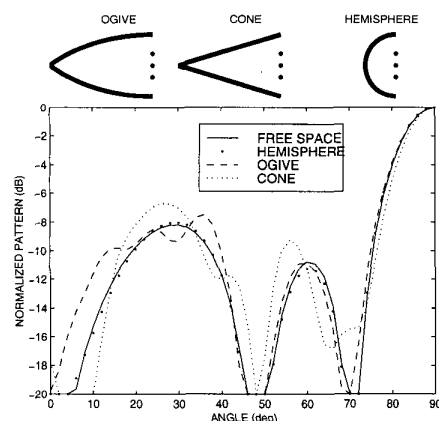


Fig. 3. The comparison of the radiation of a dipole array in the presence of three types of radome shapes: (a) Ogive shape (dashed line), (b) Straight cone shape (dotted line), and (c) Hemisphere shape (dots). The radiation of the same dipole array in free-space is also plotted for reference (Solid line). The cross view of the radome generating curves are shown on top of the plot and the dipole array is shown as a short line segment in the drawing.

V. References

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