

Electromagnetic Scattering from Material Coated PEC Objects: a Hybrid Volume and Surface Integral Equation Approach [†]

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1. Introduction

The solution to electromagnetic wave interaction with material coated objects has applications in radar cross section prediction for coated targets, printed circuit and microstrip antenna analysis. This problem can be studied using either differential equation or integral equation solvers. The finite difference time-domain (FDTD) method is a popular differential equation solver, as it is suitable for wide bandwidth modeling. However the implementation of absorbing boundary condition in FDTD requires additional meshing, thus resulting in extra memory and computational cost.

On the other hand, the frequency domain integral equation approach does not require extra mesh, and usually gives more accurate results. The typical methods in coated object scattering problems is to use surface integral equation, in which the unknown functions (electric current and magnetic current) are assumed on the interfaces between different materials and between material and conductors. Then the surface integral equation is formulated using free space Green's function. This method is suitable for homogeneous medium coating.

In the printed circuit and microstrip antenna simulation problems, the surface integral equation (SIE) is formulated using the multilayer medium Green's function. The advantage of this SIE is the reduction of the unknown domain (only over the conducting patch). This reduction of unknown domain is at the cost of evaluating the Green's function, which usually is given in terms of Sommerfeld integrals. Another disadvantage of this SIE formulation is the assumption that the layered medium is of infinite extent. The model fidelity is compromised for a finite size substrate.

Another approach to solving this problem is to use the FE-BIE (finite element boundary integral equation) approach. However, poor conditioning

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of the matrix system may occur. In this paper we propose a hybrid integral equation approach that combines the volume integral equation (VIE) and the surface integral equation to model the mixed dielectric and conducting structures. The volume integral equation is applied to the material region and the surface integral equation is enforced over the conducting surface. This results in a very general model as all the volume and surface regions are modeled properly. The advantage of this approach is that in the coated object scattering problem, the coating material can be inhomogeneous, and in the printed circuit and microstrip antenna simulation problem the substrate can be of finite size. Another advantage of this approach is the simplicity of the Green's function in both the VIE and SIE. However the additional cost here is the increase of the number of unknowns since the volume that is occupied by the dielectric material is meshed. This results in larger memory requirement and longer solution time in solving the MoM matrix equation. But this deficiency can be overcome by applying fast integral equation solvers such as the multilevel fast multipole algorithm.

In the following we first give the formulation of the problem using the method of moments (MoM), and then show numerical simulation results to demonstrate the validity of the proposed method.

2. Formulation of the hybrid integral equation

A scattering problem is used to illustrate the approach. The scatterer consists of perfectly conducting surfaces and volumetric dielectric media. Both the PEC surfaces and the material regions are of finite extent. When there is an excitation from an external source, then electrical current $\mathbf{J}_s$ will be induced on the PEC surface and volume electric current (also called polarization current) $\mathbf{J}_v$ will be induced in the dielectric region. Once the equivalent currents are found, the scattered electrical field, $\mathbf{E}^{scat}$, can be calculated as follows:

$$\mathbf{E}^{scat}(\mathbf{r}) = L_s(\mathbf{J}_s) + L_v(\mathbf{J}_v) \quad (1)$$

where L_s and L_v are integration operators defined on surface S and volume V , respectively, and they have the following expressions:

$$L_s(\mathbf{J}_s) = \int_S \bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS', \quad L_v(\mathbf{J}_v) = \int_V \bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dV'. \quad (2)$$

In order to find the equivalent currents $\mathbf{J}_s$ and $\mathbf{J}_v$, two integral equations are formulated. The first one, called surface integral equation, is formulated using the condition that the tangent component of the total electric field $\mathbf{E}$ on the PEC surface vanishes, i.e.,

$$\hat{\mathbf{t}} \cdot [L_s(\mathbf{J}_s) + L_v(\mathbf{J}_v)] = -\hat{\mathbf{t}} \cdot \mathbf{E}^{inc}, \quad (3)$$

where $\mathbf{E}^{inc}$ is the incident field (produced by the external source in the absence of the scatterer). The second integral equation, called volume integral equation, is formulated using the condition that the total electric field at any

point inside the dielectric region is the sum of the scattered field and the incident field. Since the scattered field now consists of the contribution of both surface and volume currents, the integral equation can be written as

$$\mathbf{E}^{total}(\mathbf{r}) = \mathbf{E}^{inc}(\mathbf{r}) + L_s(\mathbf{J}_s) + L_v(\mathbf{J}_v). \quad (4)$$

Note that the volume current $\mathbf{J}_v$ is related to the total electric field by the following relation:

$$\mathbf{J}_v = i\omega(\epsilon - \epsilon_b)\mathbf{E}^{total}(\mathbf{r}). \quad (5)$$

Due to the interactions between the two currents $\mathbf{J}_s$ and $\mathbf{J}_v$, the two integral equations (3) and (4) must be solved simultaneously. We use the method of moments (MoM) to solve these two integral equations. To this end, we follow the typical MoM procedure. First the surfaces are discretized into small triangles and the dielectric volume region are discretized into small tetrahedrons. Then the generalized roof-top basis functions (also called RWG basis functions [4]) are used to expand the unknown currents. The expansion takes the following forms:

$$\mathbf{J}_s = \sum_{n=1}^{N^s} a_n^s f_n^s(\mathbf{r}), \quad \epsilon_r \mathbf{J}_v = \sum_{n=1}^{N^v} a_n^v f_n^v(\mathbf{r}). \quad (6)$$

Next the approximate currents in (6) are inserted into (3) and (4) and Galerkin's test method is used to convert the two integral equations into linear algebra equation, which are then solved numerically. Note in equation (6), the quantity $\epsilon_r \mathbf{J}_v$ is approximated instead of $\mathbf{J}_v$. This treatment will ensure the continuity of the electric flux across two cells with different permittivities. It must be noted that the above is not the only way to solve integral equations. The choice of triangle/tetrahedron cells is just for the convenience of representing complex three dimensional structures. For some applications, using other discretization such as quad patch/hexahedron cells may results in significantly less number of unknowns. The solution to the linear algebra equation can be obtained either iteratively or with matrix inversion for small number of unknowns. To simulate large problems, the multilevel fast multipole algorithm may be used.

3. Numerical Results

In this section, we present a numerical example to demonstrate the validity of the theory outlined in the previous section. A coated dielectric sphere is used for this purpose since the exact solution exists for spherical layered structures. The structure of the example consists of a PEC sphere and a spherical dielectric shell co-centered with the PEC sphere. The PEC sphere has radius of 0.3λ , and the shell has inner and outer radius of 0.4λ , and 0.5λ (hence the coating thickness is 0.1λ). The permittivity of the coating dielectric is $\epsilon_r = 4$. The PEC surface is divided into 300 triangles, and the dielectric shell is divided into 900 tetrahedrons as shown in Figure 1 (for the

purpose of display the mesh for the coated dielectric part is cut into two and separated). The calculated RCS using the method described in this paper is compared with exact solution in Figure 2.

This example is intended to show the validity of the proposed approach. The size of the structure is small due to the limitation of the computer memory. Our next step is to apply the multilevel fast multipole algorithm to reduce memory requirements and the solution time.

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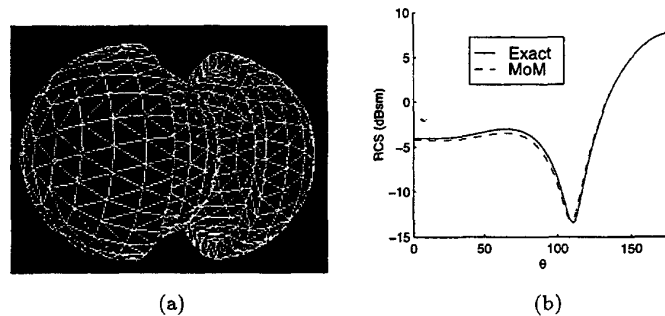


Figure 1. (a) The mesh for the coated sphere (the tetrahedron mesh for the coating material is cut into two and one half is separately displayed in order to show the interior mesh for the conducting surface. (b) The RCS of the coated sphere for vertical incident wave at $f = 300$ MHz.