

VOLUME INTEGRAL EQUATION SOLUTION OF MICROWAVE ABSORPTION AND SCATTERING BY RAINDROPS

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I. INTRODUCTION

Frequencies above 5 GHz, especially in the microwave or millimeter-wave range, used for future terrestrial repeater system or earth-satellite communication systems may cause various degrading on the performance of communication lines due to the attenuation by hydrometers (e.g., rain, hail, fog, snow, and ice crystals). In this paper, we consider only attenuation (absorption and scattering) from rainfall, since hail, fog, snow, and ice crystal are considered as secondary affects in the studies of RF signal propagation. The problem of attenuation by rain has been studies for more than four decades. In most theoretical studies, the raindrop is considered as a sphere or an oblate spheroid for the computation of rain attenuation. Indeed, raindrops are non-spherical in shape and have preferred axial orientation angles different from vertical direction. Morrison and Cross[1] use the least-squares fitting process of the boundary conditions and deal with raindrop as a spheroidal object to calculate the rain attenuation. Later, based on the spheroidal models, further developments have been achieved by many researches. A typical and well-accepted model for describing the realistic raindrop shapes was developed by Pruppacher and Pitter[2] who solved a pressure balance equation at the surface of falling raindrops by numerical techniques and determined the shapes of raindrops of various sizes theoretically. Recently, Li et al.[3] used different functional expression to further simplify the P-P model for the calculation of rain attenuation. This new modified P-P model[3] will be adopted for the studies of microwave absorption and scattering by raindrops in this paper.

II. MODIFIED P-P MODEL OF RAINDROPS

Based on the Pruppacher and Pitter's nonlinear equation[2] and cosine series function, a new modified P-P model of raindrop proposed by Li et al.[3] has been obtained. In this new model, the surface of a raindrop in three-dimensional (3-D) spherical coordinates (r, θ, ϕ) is given by [3]

$$r = a(1 - v)(1 + \frac{v}{1 - v} \sin^2 \theta) \quad (1)$$

Where $a = 1.11582 a_0$

$$v = v_1 = -1.375447 \times 10^{-2} + 6.543960 \times 10^{-2} a, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq \phi \leq 2\pi$$

$$v = v_z = -7.239211 \times 10^{-2} + 1.82756 \times 10^{-1} \theta, \quad \frac{\pi}{2} \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi$$

While a_0 denotes the raindrop mean radius. Using Eq. 1, a 3-D raindrop shape with a mean radius $a_0 = 3.5$ mm is obtained as shown in Fig. 1. From Fig. 1, it is clear that the bottom of a large raindrop looks flattened in the side view but caved-in on the cutaway view.

III. VIEF IMPLEMENTED WITH MOMENT METHOD

The volume integral equation formulation (VIEF) implemented with moment method [4] was developed to calculate the scattering and absorption by aerosol particles in previous works [5].

The internal electromagnetic fields inside a dielectric object, $\vec{E}$ and $\vec{H}$, can be summed of the incident $\vec{E}^i$ and $\vec{H}^i$ and the scattered, $\vec{E}^s$ and $\vec{H}^s$ fields. That is $\vec{E} = \vec{E}^i + \vec{E}^s$, $\vec{H} = \vec{H}^i + \vec{H}^s$ (2)

Based on Van Bladel's analysis [6], a tensor integral equation is obtained

$$\left[1 + \frac{\tau(\vec{r})}{3\epsilon_0} \right] \vec{E}(\vec{r}) - \text{PV} \int_V \tau(\vec{r}') \vec{E}(\vec{r}') \vec{G}(\vec{r}, \vec{r}') d\vec{r}' = \vec{E}^i(\vec{r}) \quad (3)$$

Where

$$\tau(\vec{r}) = \sigma(\vec{r}) + j\omega(\epsilon(\vec{r}) - \epsilon_0)$$

The PV symbol is called the "principle value" of the integral, $\vec{G}$ is a green dyadic function. σ is the conductivity of the dielectric object. and ϵ_0 are the permittivities of the dielectric object and the free space, respectively. $\vec{r}$ and $\vec{r}'$ indicate the locations of the field and source points, respectively. In Eq. 3, $\vec{E}(\vec{r})$ is the unknown total electric field inside the dielectric objects to be determined. $\vec{E}^i(\vec{r})$ is the known incident electric field. In order to solve Eq. 3 numerically for electric fields, the moment method together with a point matching procedure are then applied to convert the 3N summation into 3N simultaneous equations in the 3N unknown field components in the N mathematical cells. Then a matrix equation is then obtained. Once the internal fields are determined by solving the matrix equation, the total absorbed power in the dielectric objects and the scattered power by the dielectric objects can be calculated.

IV. NUMERICAL RESULTS

To examine the accuracy of the VIEF procedure, we compare the obtained results of total cross sections with those obtained from Morrison's solution [1] for spheroidal models of raindrops and Li's method for P-P raindrop models [2]. At 20, we take the refractive index $N_r = 5.581 - j2.848$ for the simulations of rain attenuation at 30 GHz. The total cross sections obtained by VIEF, Morrison's solution [1], and Li's method [3] are shown in Figs. 1 and 2. It can be seen from Figs. 1 and 2 that the total cross sections obtained by the VIEF using P-P models have about 5 and 7 % higher than those obtained by the VIEF using spheroidal models for horizontal and vertical polarizations,

respectively. It is also shown that the results obtained by the VIEF are much close to those obtained by the Li's method for both horizontal and vertical polarizations. While the results obtained by the VIEF and Li's method are much higher than those obtained by Morrison's solution for both horizontal and vertical polarizations, especially, in the larger raindrops. The main reason of larger relative errors between VIEF's and Morrison's results is resulting from that the second and higher-order expansion terms of the Taylor series used by the Morrison and Cross [1] to describe the spheroidal shapes have been neglected.

V. CONCLUSIONS

Although the rain attenuation by raindrops has been investigated by many researchers previously, the further investigation of scattering and absorption due to realistically distorted raindrops is still very important for wireless communications. In this paper, we use the numerical solution, the VIEF, to calculate the scattering and absorption cross sections by raindrops. The validity of the VIEF is checked by comparing the obtained and total cross section values with those obtained from Morrison's solution for spheroidal models and Li's solution for new modified P-P models. After checking the validity and accuracy of the solution procedure, we will extensively use the VIEF to calculate the scattering and absorption cross sections by raindrops in the microwave frequency range from 0.6 to 100 GHz in the near future.

ACKNOWLEDGMENTS

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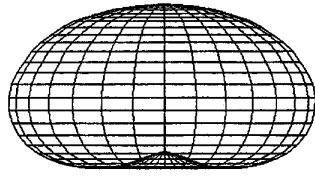


Fig. 1 Illustration of a 3-D P-P raindrop model.

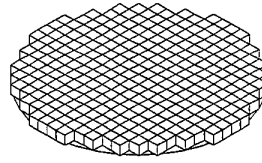


Fig. 2 Cubic cell model in one layer of a raindrop used the VIEF.

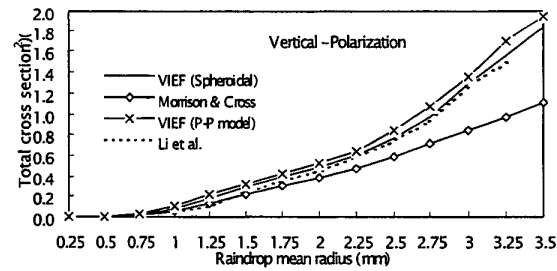


Fig. 3 Comparisons of total cross sections for vertical polarization at 30 GHz. The refractive index of raindrops is taken to be $N = 5.581 - j2.848$.

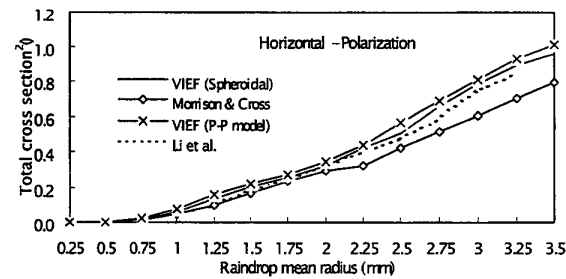


Fig. 4 Comparisons of total cross sections for horizontal polarization at 30 GHz. The refractive index of raindrops is taken to be $N = 5.581 - j2.848$.