

The Use of Huygens' Equivalence Principle for Solving 3D Volume Integral Equation of Scattering†

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1. Introduction

This paper presents an integral equation solver using the nested equivalence principle algorithm (NEPAL) which has been successfully applied to 2D problems [1]. It is known that in solving an integral equation, one can first replace the volume scatterer by small subscatterers where the size of a subscatterer is much smaller than a wavelength. The unknown function to be sought is expanded in terms of basis functions which usually have their supports on the subscatterers. By matching the field on the subscatterers, a set of linear equations are formed. The number of unknowns is proportional to the number of subscatterers in this case. Physically, each subscatterer can be considered a scattering center.

The interaction of a subscatterer with the other subscatterers can be described by interaction matrices [6]. If there are N subscatterers, then there will be N^2 interaction matrices since each subscatterer will interact with all the other subscatterers including itself. The N^2 interaction matrices can be found with N^3 operations [5,6]. The idea of NEPAL is to reduce the number of scattering centers, and hence to reduce the CPU time required for the solution, and essentially find the inverse of the integral operator with computational complexity less than $O(N^3)$.

2. Formulation of the Problem

A three dimensional volume can be discretized into a set of small cubic boxes called subscatterers. Each subscatterer has a volume Δ^3 . The scattering property of each subscatterer can be represented by a multipole with order proportional to the electrical size of the subscatterer. The scattered field due to the i -th subscatterer in isolation, located at $\mathbf{r}_i$, can be written as [2],

$$\begin{aligned} \mathbf{E}_i^{sca}(\mathbf{r}) &= \sum_{nm} \{ \mathbf{M}_{nm}(\mathbf{r} - \mathbf{r}_i) [\mathbf{b}_i^M]_{nm} + \mathbf{N}_{nm}(\mathbf{r} - \mathbf{r}_i) [\mathbf{b}_i^E]_{nm} \} \\ &= \overline{\boldsymbol{\psi}}^t(\mathbf{r} - \mathbf{r}_i) \cdot \mathbf{b}_i, \end{aligned} \quad (1)$$

† This work was supported by Office of Naval Research under grant N00014-89-J1286, the Army Research Office under contract DAAL03-91-G-0339, the National Science Foundation under grant NSF ECS 92-24466, and NASA under grant NASA NAG 2-871. The computer time was provided by the National Center for Supercomputing Applications (NCSA) at the University of Illinois, Urbana-Champaign.

where, superscripts M and E stand for TE and TM components respectively. $\bar{\psi}^t(\mathbf{r}) = [\mathbf{M}, \mathbf{N}]$ is the vector spherical wave function, $\mathbf{M}$ and $\mathbf{N}$ are given by [2,3], and $\mathbf{b}_i = [\mathbf{b}_i^M, \mathbf{b}_i^E]^t$ is the multipole coefficients which can be expressed as [4,7],

$$\mathbf{b}_i = \sum_{j=1}^{N_A} \bar{\mathbf{I}}_{ij(N_A)} \cdot \bar{\alpha}_{js} \cdot \mathbf{a}_s, \quad i = 1, 2, \dots, N_A. \quad (2)$$

Hence, the total scattered field is given by

$$\mathbf{E}^{sca}(\mathbf{r}) = \sum_{i=1}^{N_A} \bar{\psi}^t(\mathbf{r} - \mathbf{r}_i) \cdot \sum_{j=1}^{N_A} \bar{\mathbf{I}}_{ij(N_A)} \cdot \bar{\alpha}_{js} \cdot \mathbf{a}_s. \quad (3)$$

In the above, $\bar{\mathbf{I}}_{ij(N_A)}$, $i, j = 1, 2, \dots, N_A$ are called interaction matrices; they describe the interactions between N_A subscatterers. They also describe a scattering process whereby the wave enters through the j -th scattering center, and leaves through the i -th scattering center. They can be found in a variety of ways, for example, by Gaussian elimination, or by a recursive algorithm [5,6].

We can show by the Huygens' equivalence principle that there is another set of N_B interaction matrices which will generate the same scattered field such that $N_B \ll N_A$. The proof involves tedious mathematical equation, we will give a brief discussion here and the details can be found in [8].

Actually, Equation (3) provides a relation between source and scattered field. Here, the source is the induced (volume) source. Huygens' equivalence principle states that the scattered field can also be computed by integration over the equivalent sources which are distributed on the surface enclosing the volume scatterer. Thus the scattered field and the equivalent (surface) source can be related via a set of interaction matrices $\{I_{(N_B)}\}$. At this point, $\{I_{(N_B)}\}$ is not known. However, we also know from the equivalence principle that the equivalent sources are the tangential fields which are given in term of the interior (volume) source. As a result, the two sets of interaction matrices $\{I_{(N_A)}\}$ and $\{I_{(N_B)}\}$ are related and $\{I_{(N_B)}\}$ are given by [8],

$$\bar{\mathbf{I}}_{mm'(N_B)} = \sum_{i=1}^{N_A} \bar{\mathbf{h}}_{mi}^{(o)} \cdot \sum_{j=1}^{N_A} \bar{\mathbf{I}}_{ij(N_A)} \cdot \bar{\mathbf{h}}_{jm'}^{(i)}. \quad (4)$$

where the matrices $\bar{\mathbf{h}}_{jm'}^{(i)}$ and $\bar{\mathbf{h}}_{mi}^{(o)}$ are depend on the frequency and the scatterer parameters.

Now the scattered field can be calculated using the new interaction matrices,

$$\mathbf{E}^{sca}(\mathbf{r}) = \sum_{m=1}^{N_B} \bar{\psi}^t(\mathbf{r} - \mathbf{r}_m) \cdot \sum_{m'=1}^{N_B} \bar{\mathbf{I}}_{mm'(N_B)} \cdot \bar{\alpha}_{m's} \cdot \mathbf{a}_s, \quad (5)$$

As mentioned in the introduction of this paper, the idea of NEPAL is to reduce the number of scattering centers, or the number of interaction

matrices, as shown by Equation (4). Now we will describe the steps involved in this algorithm.

To begin, we define a nested group systems which consists of different levels of groups, i.e., a level l group contains eight member, each member itself is a group of level $l - 1$, where, $l = L, L - 1, \dots, 2, 1$.

We start from level 2. In each level, we find the scattering solution for each of the groups in this level, and then use Huygens' equivalence principle (Equation (4)) to replace the interior subscatterers of a group by subscatterers on the boundary of the group, so that the scattering properties of the group are represented by the subscatterers on the boundary. As a result, the interior subscatterers in each group are "removed" and the number of subscatterers in each group is reduced as shown in Figure 1. This process continues until the highest level is reached where the total number of unknowns is proportional to the number of surface unknowns of the scatterer. Using similar analysis as presented for the 2D case, the computational complexity can be shown to be CN^2 where N is the number of subscatterers and C is a constant.

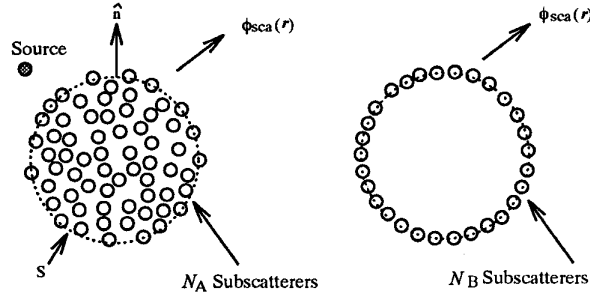


Figure 1 The number of subscatterers is reduced by removing the interior subscatterers via Huygens' equivalence principle.

3. Numerical results and conclusions

We have presented in this paper the extension of the nested equivalence principle algorithm (NEPAL) to three dimensions. One numerical result is shown in Figure 2. The algorithm is based on Huygens' equivalence principle by nesting small algorithms within a larger one. Therefore, the key element is to divide the computation into several stages and reduce the number of unknowns at each stage. This represents an efficient algorithm for directly solving the integral equation of scattering with reduced computational complexity of $O(N^2)$. In other words, this method seeks the inverse of the integral operator in less than $O(N^3)$ operations and the memory requirement is of $O(N^{1.5})$. Hence, it can be used to compute the scattering solution of large objects for many incident waves.

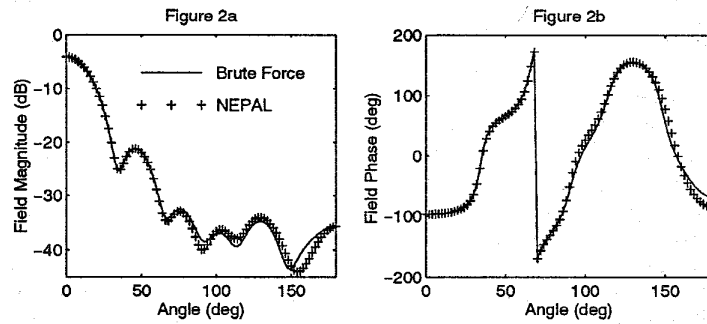


Figure 2 Scattered field of a cubic dielectric scatterer for plane wave incidence at $\theta = 180^\circ$. Cube side length = 1.6λ , $\epsilon_r = 2.6$, $f = 300(MHz)$. Observation distance $r = 10\lambda$. The cube is divided into 512 small subscatterers. (a). Magnitude; (b). Phase.

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