

Comparison of SIE-MoM and VIE-MoM for Determination of Complex Resonant Frequency of Dielectric Resonators

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Abstract — Volume Integral Equation (VIE) and Surface Integral Equation (SIE) formulations of the resonant mode, frequency, and quality factor of arbitrarily shaped Dielectric Resonators (DR) in free space and semi-open multilayered medium are compared. Although both methods are very efficient tools for resonant field/frequency determination for microwave circuit design and analysis, we have found that SIE is not suitable for determining the quality factor of high quality DR's of interest in high performance microwave filters and oscillators. As the quality factor goes beyond several hundred, the quality factor obtained through SIE-MoM becomes erroneous. It may fluctuate randomly depending on the mesh. However, VIE-MoM has been found to be robust and reliable for DR's with very high quality factors.

1. INTRODUCTION

Dielectric resonator (DR) is a common component in microwave circuit design, which can be used in oscillators or filters or as antennas [1]. Two important parameters of DR are its resonant frequency and quality factor. Also knowing the field distribution of resonant mode may help the designer to design the coupling structures, therefore a fast and accurate simulator for determining the resonant modes of DR is desired. Another requirement, which has been particularly emphasized by some recent innovative structures, is the capability to simulate arbitrarily shaped DR's such as cubic structures with cut corners and/or those which include slots or notches.

Method of Moments (MoM) is a favorable numerical method for simulating DR in free space or multilayered medium due to its efficiency and versatility. The problem of DR can be formulated either by volume integral equation (VIE) [2-4] or surface integral equation (SIE) [5-6], which is then solved by MoM. VIE is based on volume equivalence theorem which takes the volume of the DR as its solution domain, while SIE is based on surface equivalence theorem which takes the surface of the DR as its solution domain, therefore generally SIE-MoM results in fewer unknowns and less matrix-filling time than VIE-MoM.

However, based on our comparative study and analysis of SIE-MoM and VIE-MoM for determining the resonant modes of DR in free space and semi-open multilayered medium we found an important shortcoming of SIE-MoM in predicting the quality factor of DR. This had not been reported earlier. Different from DR in metal enclosure or waveguide, the quality factor of which is mainly due to metal or dielectric loss and its value is generally above thousands, the quality factor of DR in free space or semi-open multilayered medium is mainly due to radiation and its value ranges from tens to hundreds. It is found SIE-MoM computational scheme for quality factor becomes numerically unstable when the quality factor is greater than several hundred, while VIE-MoM remains much more robust.

The outline of the paper is as follows: Section I is the introduction. Section II briefly describes the formulation of VIE-MoM and

TABLE I
DEPENDENCE OF Q ON THE LOSS OF A RECTANGULAR DR IN FREE SPACE. (14.98x14.98x7.48mm)

Dielectric constant	Method and mesh	Re(f)	Im(f)	Quality factor
200-j20	VIE 4x4x1	1.37304	0.0693236	9.90312
	SIE 4x4x2	1.36967	0.0687629	9.95939
200-j2	VIE 4x4x1	1.37721	0.00836787	82.2913
	SIE 4x4x2	1.37497	0.00800538	85.878
200-j1	VIE 4x4x1	1.37727	0.0050036	137.628
	SIE 4x4x2	1.375	0.0046411	148.133
200-j0.2	VIE 4x4x1	1.37732	0.00226862	303.558
	SIE 4x4x2	1.375	0.00186336	368.957
	SIE 5x5x2	1.3778	-0.00281238	-244.953
200-j0	VIE 4x4x1	1.37731	0.00157013	438.597
	VIE 4x4x2	1.40276	0.00179412	390.932
	SIE 3x3x2	1.36865	0.00321763	212.68
	SIE 4x4x2	1.375	0.00120764	569.295
	SIE 5x5x2	1.37782	-0.00346809	-198.642
	SIE 6x6x2	1.37955	-0.00570212	-120.969
	SIE 4x4x3	1.3787	-0.0054781	-125.837

SIE-MoM. Section III gives the numerical examples. Section IV provides some concluding remarks.

II. FORMULATIONS

A. VIE-MoM

Denote the space occupied by the DR as V , and the permittivity of the DR is $\epsilon_r(\mathbf{r})$. Assuming time dependence of $e^{j\omega t}$, the VIE can be set up as

$$\begin{aligned} \hat{E}(\mathbf{r}) = & -j\omega \int_V \bar{G}^e(\mathbf{r}|\mathbf{r}') \bar{J}_e(\mathbf{r}') dV' \\ & - \nabla \int_V G^s(\mathbf{r}|\mathbf{r}') \rho_s(\mathbf{r}') dV' \end{aligned} \quad (1)$$

where

$$\bar{J}_e(\mathbf{r}) = j\omega[\epsilon_r(\mathbf{r}) - \epsilon_0] \bar{E}(\mathbf{r}) \quad (2)$$

is the equivalent volume current. ϵ_r is the permittivity of the medium in which the DR is embedded. $\bar{G}^e$ and G^s are the Green's functions for the vector and scalar potential, respectively.

The tetrahedral basis function proposed by Schaubert [7] is used to expand $\bar{J}_e$. Galerkin's procedure is then applied to (1) to obtain the matrix equation

$$[Z]I = 0 \quad (3)$$

B. SIE-MoM

Denote the surface of the DR as S . For the convenience of representation, the permittivity and permeability inside the DR are denoted as ϵ^+ and μ^+ , while the permittivity and permeability of the surrounding medium is denoted as ϵ^- and μ^- , as shown in Fig.1. There are different ways to formulate the SIE. For simplicity, the EFIE's are used here:

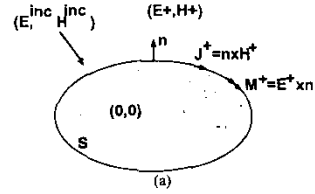


TABLE II
DEPENDENCE OF Q ON THE LOSS OF THE DR ON A GROUNDED SUBSTRATE.
(14.98x14.98x7.48mm)

Dielectric constant	Method and mesh	Re(f)	Im(f)	Quality factor
34.19-3j	VIE 4x4x2	3.68235	0.130049	14.1575
	SIE 4x4x2	3.669	0.151251	12.1289
	SIE 5x5x2	3.67833	0.13966	13.1689
	SIE 4x4x3	3.6845	0.125757	14.6493
34.19-0.3j	SIE 4x4x2	3.68015	0.0106711	172.435
	SIE 4x4x3	3.69503	-0.0160454	-115.143

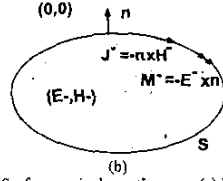


Fig. 1 Surface equivalence theorem. (a) Exterior problem (b) Interior problem.

obtained by searching the roots of $|Z|$, as is the case for VIE-MoM. However, we found that this method cannot be applied to SIE-MoM. The details are explained in [6]. Instead, the minimum of the reciprocal of the condition number of impedance matrix is searched.

Once the complex resonant frequency is found, the quality factor is calculated through

$$Q = \frac{f_r}{\Delta f_{3dB}} \quad (10)$$

$$\vec{E}^-(\vec{r}) = -j\omega\vec{A}^-(\vec{r}) - \nabla\Phi^-(\vec{r}) - \frac{1}{\epsilon} \nabla \times \vec{F}^-(\vec{r}) \quad (4)$$

$$\vec{E}^-(\vec{r}) = -j\omega\vec{A}^-(\vec{r}) - \nabla\Phi^-(\vec{r}) - \frac{1}{\epsilon} \nabla \times \vec{F}^-(\vec{r}) \quad (5)$$

where

$$\vec{A}^-(\vec{r}) = \int_V \vec{G}^-(\vec{r}|\vec{r}') \cdot \vec{J}_s^-(\vec{r}') d\vec{r}' \quad (6)$$

$$\Phi^-(\vec{r}) = \int_V G^-(\vec{r}|\vec{r}') \rho_s^-(\vec{r}') d\vec{r}' \quad (7)$$

$$\vec{F}^-(\vec{r}) = \int_V \vec{G}^-(\vec{r}|\vec{r}') \cdot \vec{M}_s^-(\vec{r}') d\vec{r}' \quad (8)$$

$$\rho_s^-(\vec{r}) = -\frac{1}{j\omega} \nabla \cdot \vec{J}_s^-(\vec{r}) \quad (9)$$

The same set of RWG basis function [8] is used to expand both electric and magnetic current. However, two sets of testing functions are used for (4) and (5), one is the same as the basis functions, the other is the basis function rotated by 90 degree around the normal vector [6]. Once the testing procedure has been applied, a matrix equation with similar form as (3) is obtained.

C. Determination of Resonant Frequency and Quality Factor

Eq.(3) has non-zero solutions only at complex resonant frequencies, where the impedance matrix becomes singular. Generally the complex resonant frequency is

III. NUMERICAL RESULTS

First we consider rectangular DR's with the same dimensions, high dielectric constant, but with different loss tangent. Such DR's will have similar resonant frequencies but different quality factor due to loss. The purpose of using high dielectric constant is to decrease radiation loss so that the quality factor can be controlled through varying the loss tangent. Table I shows the simulation results using VIE and SIE and different mesh sizes. The mesh size is denoted by 3 numbers. These numbers are the number of segments along the three orthogonal axes. An example of which is shown in Fig.2. From Table.I we can see that the calculated resonant frequencies agree quite well. When the quality factor is less than about 150, the quality factor calculated by SIE and VIE are very close. However, when the quality factor is above about 150, the quality factor obtained by SIE becomes stable. For some meshes it is positive while for other mesh configuration it might even become negative.

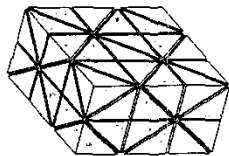


Fig. 2. A $3 \times 3 \times 2$ mesh for a rectangular DR.

As a second example, a rectangular DR on a grounded substrate is considered. The dielectric constant of the substrate is 9.8 and its thickness is 0.9mm. The results are listed in Table II, from which we can see that the real parts of the resonant frequencies agree quite well, while the quality factor calculated by SIE becomes unstable when the quality factor becomes large.

The main reason for this behavior, although it is still under investigation, is believed to be the stronger variational nature and the weaker singularities of the VIE-MoM procedure compared to the other formulation.

IV. CONCLUSIONS

In this paper VIE-MoM and SIE-MoM are compared for determining the resonant frequency of DR. It is found both of them can get accurate result for real part of the resonant frequency. However, the imaginary part of the resonant frequency obtained by SIE-MoM becomes not unreliable when the quality factor of the DR is high, which causes the quality factor obtained by SIE-MoM numerically unstable.

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REFERENCES

- [1] D. Kajfez and P. Guillon, Eds., *Dielectric Resonators*. Norwood, MA: Artech, 1986.
- [2] S.-L. Lin and G. W. Hanson "An efficient full-wave method for analysis of dielectric

resonators possessing separable geometries immersed in inhomogeneous environments," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-48, no. 1, pp. 84-92, Jan. 2000.

- [3] S.-Y. Ke and Y.-T. Cheng, "Integration equation analysis on resonant frequencies and quality factors of rectangular dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-49, pp. 571-574, Mar. 2001.
- [4] Y. Liu, S. Safavi-Naeini, S. K. Chaudhuri and R. Sabry, "Simulation of resonant modes of rectangular DR in MIC environment using MPIE-MoM with combined entire-domain and sub-domain basis functions," *2002 IEEE MTT-S International Microwave Symposium Digest*, Seattle, 2002, vol.3, pp.1901-1904
- [5] A. W. Glisson, D. Kajfez, and J. James, "Evaluation of mode in dielectric resonators using a surface integral equation formulation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 1023-1029, Dec. 1983.
- [6] Y. Liu, S. Safavi-Naeini, S. K. Chaudhuri and R. Sabry, "On the determination of resonant modes of dielectric objects using surface integral equations," submitted to *IEEE Trans. Antennas Propagat.*
- [7] D. H. Schaubert, D. R. Wilton, and A. W. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 77-85, Jan. 1984.
- [8] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 409-418, May 1982.