

Scattering by buried dielectric objects: Volumetric Integral Equation approach

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Abstract — The application of the Volumetric Integral Equation (VIE) method to the solution of the problem of scattering from a three-dimensional object buried in a dielectric half-space is outlined. The VIE method described and applied in this paper is basically an extension of that proposed by Richmond and originally applied to dielectric bodies in free space. The approximate solution of the integral equation for the total electric field is performed by discretizing the dielectric body and reducing the equation itself to a linear system. The computation of the coefficients of this latter system involves the evaluation and multiple integration of the suitable dyadic Green's function. Through some analytical considerations and the use of efficient automatic integrators, a computational time reduction is obtained.

INTRODUCTION

The scattering from dielectric objects buried underground has a noticeable interest for applications, such as in biomedical imaging, remote sensing, geological survey.

In this paper, the application of the Volumetric Integral Equation (VIE) method to the scattering from buried dielectric bodies (3D problem) is presented. This method can be used when the object dimensions are comparable with the source wavelength and is particularly suitable to analyze those cases in which the buried body is strongly inhomogeneous.

The method proposed is an extension [1, 2] of the original Richmond method [3] and Livesay and Chen [4] applied it successfully to study field distributions inside three-dimensional bodies in free space. Other applications to 3D scattering problems can be found in [5, 6, 7], where a tetrahedral partition of the interior volume of the body

is generally adopted.

The application of VIE method can result particularly useful in inverse scattering problems, as in remote sensing or noninvasive techniques for biological imaging.

FORMULATION

The 3D-problem under analysis is depicted in Fig. 1. A plane interface divides the space into two half-spaces, of which one is empty and the other filled with a homogeneous dielectric, having a dielectric permittivity ϵ_1 . A dielectric body C , with dielectric permittivity ϵ_2 , is completely buried into the dielectric medium. The incident field is produced by a source located in the empty half-space. The sources of practical interest (as pencil beams) can be developed in plane waves or elementary electric dipoles. Therefore, both a TM plane wave (having null magnetic field component along the interface) and an elementary dipole (vertical or horizontal with respect to the interface) are considered.

An orthogonal coordinate system is chosen, having the xy -plane coincident with the interface between the two media. The choice of the origin of the system and of the orientation of the x -axis depends upon the source under consideration. An $e^{j\omega t}$ time dependence is assumed and not explicitly indicated in the following. The dielectric body is supposed to have a generic shape and to be possibly inhomogeneous, so that its dielectric permittivity is generally a function of the position vector $\mathbf{r}$: $\epsilon_2 = \epsilon_2(\mathbf{r})$. Furthermore, losses can be present in the interior region of the body, so that ϵ_2 may be complex with a negative imaginary part.

The VIE method [3, 4] is very suitable for applications in inverse scattering problems. A discretization of the body volume is performed by using parallelepiped elements. This type of discretization causes limitations in properly approximating the body shape, but, on the other

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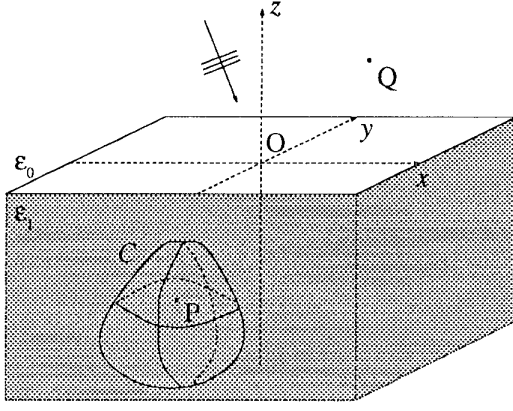


Figure 1: Geometry of the problem.

hand, it allows a reduction of the computational work by using efficient automatic integrators and reducing the number of integral evaluations required.

INTEGRAL EQUATION

The main steps of VIE method application are outlined next. An integral equation for electric field is obtained. Then this integral equation is numerically discretized, reducing its solution to the inversion of a linear system of algebraic equations.

The total electric field $\mathbf{E}$ at any point of space can be seen as the superposition of the incident field $\mathbf{E}^i$ and the scattered field $\mathbf{E}^s$. The scattered field is generated by polarization currents, existing only inside the dielectric body. Their density $\mathbf{J}(\mathbf{r})$ is related to the total field by the following expression:

$$\mathbf{J}(\mathbf{r}') = j\omega [\varepsilon_2(\mathbf{r}') - \varepsilon_1] \mathbf{E}(\mathbf{r}'), \quad (1)$$

where $\mathbf{r}'$ is the position vector of a generic point P internal to the body. The expression of the scattered field, as quoted in [4], is:

$$\mathbf{E}^s(\mathbf{r}) = \iiint_V \mathcal{G}(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') d\mathbf{r}' - \frac{\mathbf{J}(\mathbf{r})}{3j\omega\varepsilon_1}. \quad (2)$$

The integral is evaluated on the volume V of the body and must be considered as its principal value. Besides, $\mathcal{G}(\mathbf{r}, \mathbf{r}')$ is the dyadic Green's function of the problem, defined as a 3×3 -matrix, whose j th-column ($j = 1, 2, 3$) gives the field at $\mathbf{r}$ generated by a unit electric dipole located at $\mathbf{r}'$ and oriented as the correspondent axis of the orthogonal reference system.

By using eqs. (1) and (2) and by observing that $\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^s(\mathbf{r})$, the following integral equation is derived:

$$\frac{1}{3} \left[\frac{\varepsilon_2(\mathbf{r})}{\varepsilon_1} + 2 \right] \mathbf{E}(\mathbf{r}) - j\omega \iiint_V [\varepsilon_2(\mathbf{r}') - \varepsilon_1] \mathcal{G}(\mathbf{r}, \mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}' = \mathbf{E}^i(\mathbf{r}). \quad (3)$$

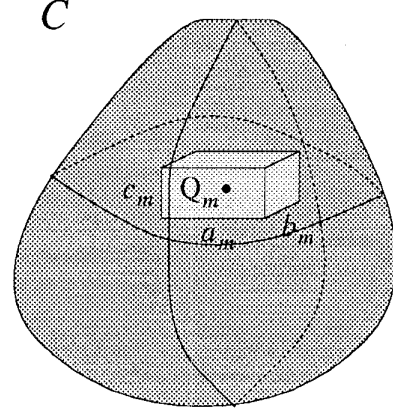


Figure 2: Discretization of the buried body in VIE method.

NUMERICAL SOLUTION

In order to find a numerical solution for eq. (3), the body volume is divided into N small cells, that can be assumed to be parallelepipeds, with centers at $Q_m \equiv \mathbf{r}_m$ and sides a_m, b_m, c_m ($m = 1, \dots, N$), as shown in Fig. 2. Inside the m -th cell, if small enough, the value of the integrand at a generic position $\mathbf{r}'$ can be assumed to be constant and equal to its value at $\mathbf{r}_m$:

$$[\varepsilon_2(\mathbf{r}') - \varepsilon_1] \mathbf{E}(\mathbf{r}') \approx [\varepsilon_2(\mathbf{r}_m) - \varepsilon_1] \mathbf{E}(\mathbf{r}_m).$$

Therefore, the integral equation (3) is approximated by the following linear system of equations:

$$\frac{1}{3} \left(\frac{\varepsilon_{2,m}}{\varepsilon_1} + 2 \right) \mathbf{E}_m - j\omega \sum_{n=1}^N [\varepsilon_{2,n} - \varepsilon_1] C_{mn} \mathbf{E}_n = \mathbf{E}_m^i, \quad (4)$$

with $m = 1, \dots, N$ and:

$$C_{mn} \triangleq \iiint_n \mathcal{G}(\mathbf{r}_m, \mathbf{r}') d\mathbf{r}'. \quad (5)$$

The matrices C_{mn} depend on the dyadic Green's function for an electric dipole in the presence of a plane interface between two distinct media. By using Hertz potential method [8, 9], this dyadic Green's function can be expressed as:

$$\mathcal{G}(\mathbf{r}, \mathbf{r}') = -j\omega\mu_0 \left[\mathcal{I} + \frac{\nabla \nabla}{k^2} \right] \begin{bmatrix} \Pi' & 0 & 0 \\ 0 & \Pi' & 0 \\ \Pi'' & \Pi'' & \Pi''' \end{bmatrix}, \quad (6)$$

where $\mathcal{I}$ is the 3×3 identity matrix, k is the wavenumber of the medium containing the observation point $\mathbf{r}'$ and the columns of the matrix on the right side are the Hertz potentials for x - and y -horizontal and z -vertical dipoles in the presence of the interface, respectively.

The expressions for Π', Π'', Π''' can be sketched as:

$$\Pi^i(x, y, z; x'y', z') = \begin{cases} \bar{\Pi}^i(x - x', y - y', z, -z'; k_0, k_1) & \text{if } z' \leq 0 \\ \bar{\Pi}^i(x' - x, y' - y, -z, z'; k_1, k_0) & \text{if } z' \geq 0 \end{cases}, \quad (7)$$

with $l = ', ', ''$ and:

$$\begin{aligned} \bar{\Pi}'(\xi, \eta, \zeta, h; k_0, k_1) = & \begin{cases} \int_{-\infty}^{+\infty} p'(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) - \tau_0 \zeta - \tau_1 h]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \geq 0, \\ \frac{\exp(jk_1 d)}{4\pi d} - \frac{\exp(jk_1 d')}{4\pi d'} + \\ + \int_{-\infty}^{+\infty} q'(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) + \tau_1(\zeta - h)]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \leq 0; \end{cases} \quad (8) \end{aligned}$$

$$\begin{aligned} \bar{\Pi}''(\xi, \eta, \zeta, h; k_0, k_1) = & \begin{cases} \int_{-\infty}^{+\infty} p''(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) - \tau_0 \zeta - \tau_1 h]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \geq 0, \\ \int_{-\infty}^{+\infty} q''(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) + \tau_1(\zeta - h)]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \leq 0; \end{cases} \quad (9) \end{aligned}$$

$$\begin{aligned} \bar{\Pi}'''(\xi, \eta, \zeta, h; k_0, k_1) = & \begin{cases} \int_{-\infty}^{+\infty} p'''(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) - \tau_0 \zeta - \tau_1 h]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \geq 0, \\ \frac{\exp(jk_1 d)}{4\pi d} + \frac{\exp(jk_1 d')}{4\pi d'} + \\ + \int_{-\infty}^{+\infty} q'''(\cdot) \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) + \tau_1(\zeta - h)]\} \cdot \\ \cdot d\alpha_\xi d\alpha_\eta & \text{if } \zeta \leq 0; \end{cases} \quad (10) \end{aligned}$$

where k_i , $i = 0, 1$ are the wave numbers of the two half-spaces, $\tau_i = \sqrt{k_i^2 - (2\pi)^2(\alpha_\xi^2 + \alpha_\eta^2)}$, $i = 1, 2$, $d = d(h) = \sqrt{\xi^2 + \eta^2 + (\zeta + h)^2}$, $d' = d(-h)$ and finally p^l, q^l , $l = ', ', ''$ are known functions of τ_0, τ_1, k_0, k_1 [8, 9].

It can be observed that Hertzian vectors are given by the free-space expression augmented by the image contribution (if present) and integral terms. The image contribution can be also written in integral form as:

$$\frac{\exp(jkd')}{4\pi d'} = \int_{-\infty}^{+\infty} \frac{1}{2j\tau} \exp\{j[2\pi(\alpha_\xi \xi + \alpha_\eta \eta) + \tau_1(\zeta - h)]\} \cdot d\alpha_\xi d\alpha_\eta \quad (11)$$

The free-space contribution to the matrices C_{mn} is discussed in [4]. The other contributions are given by expressions similar to those in eq. (9), with $\zeta - h < 0$. Therefore, by exchanging the volume integral with the improper one, the exponentials can be exactly integrated with respect to the space coordinates and this reduces the number of integrals to be evaluated from five to two (this fact is also underlined for 2D problems in [1]). By using efficient automatic integrators, the evaluation time can be further reduced.

CONCLUSIONS

An application of Volumetric Integral Equation (VIE) method for analyzing the scattering from three-dimensional objects buried into a dielectric half-space has been described. This technique allows to take into account of the presence of the interface by using the dyadic Green's function for an elementary electric dipole in the presence of an interface.

The advantages of using such a method in inverse problems, that are of wide interest in remote sensing (earth science, geological surveys), have been underlined. Moreover, the number of integrations required to evaluate the matrix entries is reduced by applying the technique presented. As a shortcoming, it is important to note that VIE method involves dense and asymmetric matrices, so that particular care must be put in their inversion.

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