

RCS Computation of Large Inhomogeneous Objects Using a Fast Integral Equation Solver

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Abstract—In this paper, we apply a fast Fourier transform (FFT) accelerated volume integral equation solver to compute the radar cross section of large-scale inhomogeneous objects. This method is related to Bojarski's k -space method and the subsequent conjugate-(CG) and biconjugate-gradient (BCG) FFT algorithms. The method developed here combines the weak-form discretization with the BCG and stabilized BCG (BCGS) solvers. We show that this method has marked improvements over related algorithms. The numerical method has been validated by Mie series for multilayer spheres and applied to some practical problems. With this method we are currently able to solve a three-dimensional problem with a volume of size $3648\lambda^3$ (21.23 million unknowns) on a single workstation.

Index Terms—Biconjugate-gradient-fast Fourier transform (BCG-FFT), scattering, spectral method, stabilized BCG (BCGS)-FFT, volume integral equation.

I. INTRODUCTION

THE SCATTERING of electromagnetic waves by penetrable, inhomogeneous objects is an important research subject and has been under investigation for over three decades [1], [2] in computational electromagnetics. From the far-zone scattered electromagnetic field, the scattering strength of the object is expressed as the so-called radar cross section (RCS). Our objective in this paper is to solve large-scale problems involving inhomogeneous dielectric objects.

As illustrated in Fig. 1, the object of interest can have an arbitrary shape and can have arbitrary space-dependent dielectric constant $\epsilon_r(\mathbf{r})$ and conductivity $\sigma(\mathbf{r})$. From the incident electromagnetic waves, we aim to find the scattered electromagnetic field and, thus, the RCS.

Since the object has an arbitrary inhomogeneity, surface integral equation methods which are amenable to the recent fast multipole (FMM) acceleration are, in general, not appropriate for this application [3]–[6]. Although one special case of this problem, i.e., an inhomogeneous object with piecewise homogeneous subdomains, can be treated by surface integral equation methods, the efficiency decreases as the number of subdomains increases. For general inhomogeneous objects, the volume integral equation methods are more suitable. Other alternative frequency domain methods include finite-element methods. In this work, we will focus on the volume integral equation methods

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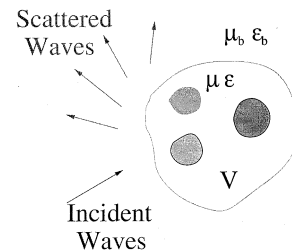


Fig. 1. Geometry of the scattering by an inhomogeneous object.

for large-scale problems whose volume can be as large as thousands of λ^3 (λ is the shortest wavelength).

One conventional method of solution for the volume integral equation is the method of moments (MoM) [7]. In the MoM, the integral equation is discretized into a linear algebraic equation with N unknowns. The direct inversion of such a matrix equation requires $O(N^3)$ CPU time and $O(N^2)$ memory. The CPU time can be reduced to $O(KN^2)$ by using iterative methods such as the conjugate-gradient method, where K is the number of iterations. However, both direct and iterative MoM methods are limited to relatively small problems because of their excessive CPU time and memory requirements.

In the early 1980s, Bojarski proposed the k -space method [9]. The k -space method uses an iterative approach (similar to Neumann series) to solve the integral equation without explicitly forming the matrix equation. In each iteration, the matrix-vector multiply is accomplished by using the fast Fourier transform (FFT) algorithm because the volume integral is convolutional. The conjugate-gradient (CG)-FFT method was later proposed to improve the convergence speed, where the iteration is accomplished by the conjugate-gradient method [10]–[20]. The three-dimensional (3-D) CG-FFT and related methods have been reported by many researchers, for examples [15]–[25]. For a more complete review, the reader is referred to [21].

In this paper, we apply a spectral method using the FFT acceleration in the weak-form discretization of the electric field integral equation. We choose the weak-form discretization proposed by Zwamborn and van den Burg [22] because of its reduced singularity compared with other discretization scheme. In contrast to [22], however, we apply the biconjugate-gradient (BCG) method [25]–[29] and the stabilized BCG (BiCGSTAB, or BCGS for short) method [30] to further improve the convergence speed by up to a factor of 14 compared to the CG-FFT method. This spectral method requires $O(N \log_2 N)$ CPU time and $O(N)$ computer memory, where N is the number of unknowns. With this method, we are able to solve a large problem with over 21 million unknowns on a single workstation. We have validated our numerical results by comparing with Mie series

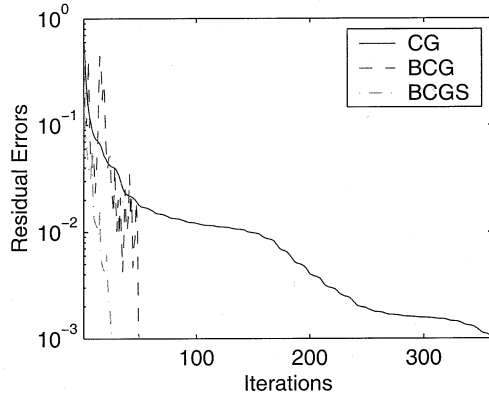


Fig. 2. Convergence curves for CG-, BCG-, and BCGS-FFT methods for a two-layer sphere.

for a multilayer spherical medium. The method has been used to model a large penetrable missile-like object.

II. FORMULATION

The objective of this work is to solve for electromagnetic fields scattered by an *inhomogeneous* object. Suppose an inhomogeneous object with a finite volume V is embedded in an isotropic, homogeneous background medium with constant permittivity ϵ_b , electric conductivity σ_b , and permeability μ_b . This inhomogeneous lossy object has spatially variable permittivity $\epsilon(\mathbf{r})$ and electric conductivity $\sigma(\mathbf{r})$ and a constant permeability $\mu = \mu_b$. The complex permittivity values for the object and background are $\tilde{\epsilon} = \epsilon - j\sigma/\omega$ and $\tilde{\epsilon}_b = \epsilon_b - j\sigma_b/\omega$, respectively. The electric field integral equation (EFIE) in the inhomogeneity is

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{inc} + \int_V \mathbf{G}(\mathbf{r} - \mathbf{r}') \cdot k_b^2 \chi(\mathbf{r}') \frac{\tilde{\epsilon}}{\tilde{\epsilon}_b} \mathbf{E}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} \in V \quad (1)$$

where $\chi = 1 - \tilde{\epsilon}_b/\tilde{\epsilon}$, $k_b^2 = \omega^2 \mu \epsilon_b - j\omega \mu \sigma_b$ is a complex contrast function, $\mathbf{G} = (\mathbf{I} + (\nabla \nabla / k_b^2))g(\mathbf{r} - \mathbf{r}')$ is the dyadic Green's function, and $g(\mathbf{r} - \mathbf{r}')$ is the scalar Green's function of the homogeneous background medium. Equation (1) is valid everywhere in space, both inside and outside the volume V . We aim to find $\mathbf{E}$ for $\mathbf{r} \in V$, from which the electric field everywhere can be found from (1).

Equation (1) is a hypersingular integral equation for $\mathbf{E}$ (Fredholm integral equation of the second kind, or Lippmann-Schwinger integral equation). Defining a magnetic vector potential $\mathbf{A}$ as

$$\mathbf{A} = \int_V g(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') \tilde{\mathbf{D}}(\mathbf{r}') d\mathbf{r}' \equiv \mathcal{A}[\tilde{\mathbf{D}}] \quad (2)$$

where $\tilde{\mathbf{D}} = \tilde{\epsilon} \mathbf{E} / \tilde{\epsilon}_b$ is the "normalized" electric flux density, the integral equation in (1) can be rewritten as

$$\frac{\tilde{\epsilon}_b}{\tilde{\epsilon}(\mathbf{r})} \tilde{\mathbf{D}}(\mathbf{r}) - k_b^2 \mathcal{A}[\tilde{\mathbf{D}}] - \nabla \left\{ \nabla \cdot \mathcal{A}[\tilde{\mathbf{D}}] \right\} = \mathbf{E}^{inc}(\mathbf{r}). \quad (3)$$

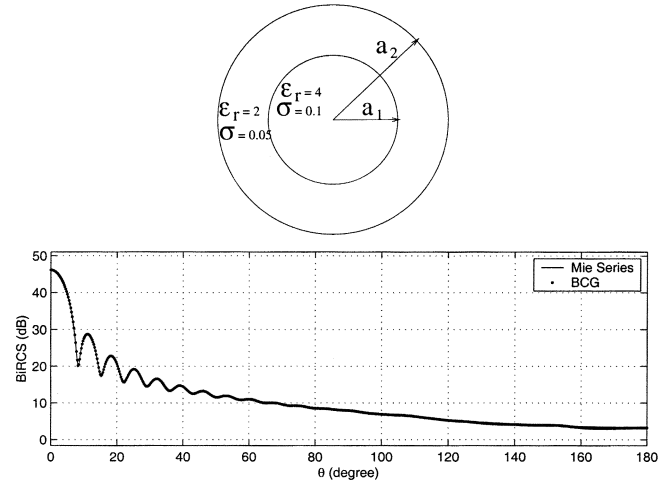


Fig. 3. Bistatic RCS as a function of θ for $\phi = \pi/2$ using weak-form BCG-FFT and Mie series at $f = 1$ GHz. The two-layer spherical medium has parameters $\epsilon_{r;1} = 4$, $\sigma_1 = 0.1$ S/m, and $a_1 = 1.0575$ m; $\epsilon_{r;2} = 2$, $\sigma_2 = 0.05$ S/m, and $a_2 = 1.2075$ m. The problem is discretized into 12.75 million unknowns, requiring 1.1-GB memory.

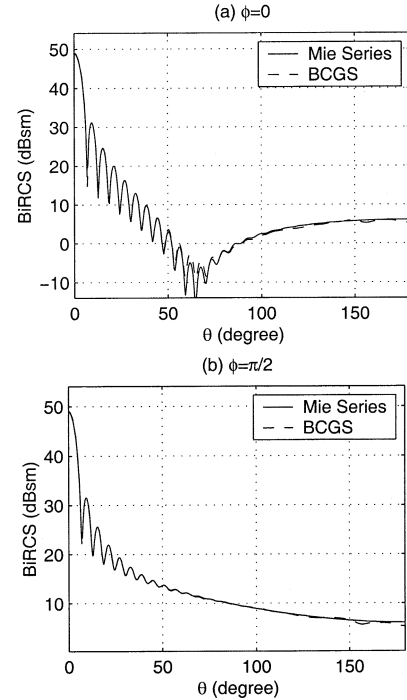


Fig. 4. RCS of the four-layer sphere at 1 GHz (21.23 million unknowns, 1.99-GB memory).

Equation (3) is also a hypersingular integral equation for the electric flux density $\tilde{\mathbf{D}} = \tilde{\epsilon} \mathbf{E} / \tilde{\epsilon}_b$. The hypersingularity arises from second-order derivatives of the scalar Green's function. Among various approaches to treat this hypersingular problem, the weak-form discretization [22] appears very attractive and will be used in this work.

Suppose the scatterer domain is embedded in a rectangular box with a dimension $L_x \times L_y \times L_z$. We discretize this box into $N_x \times N_y \times N_z$ uniform rectangular prism cells where the cells centered at $\mathbf{r}_{m,n,p} = (x_m, y_n, z_p)$ ($1 \leq m \leq N_x$, $1 \leq n \leq N_y$, $1 \leq p \leq N_z$). To solve (3) for $\tilde{\mathbf{D}}$, we first expand both $\tilde{\mathbf{D}}$ and

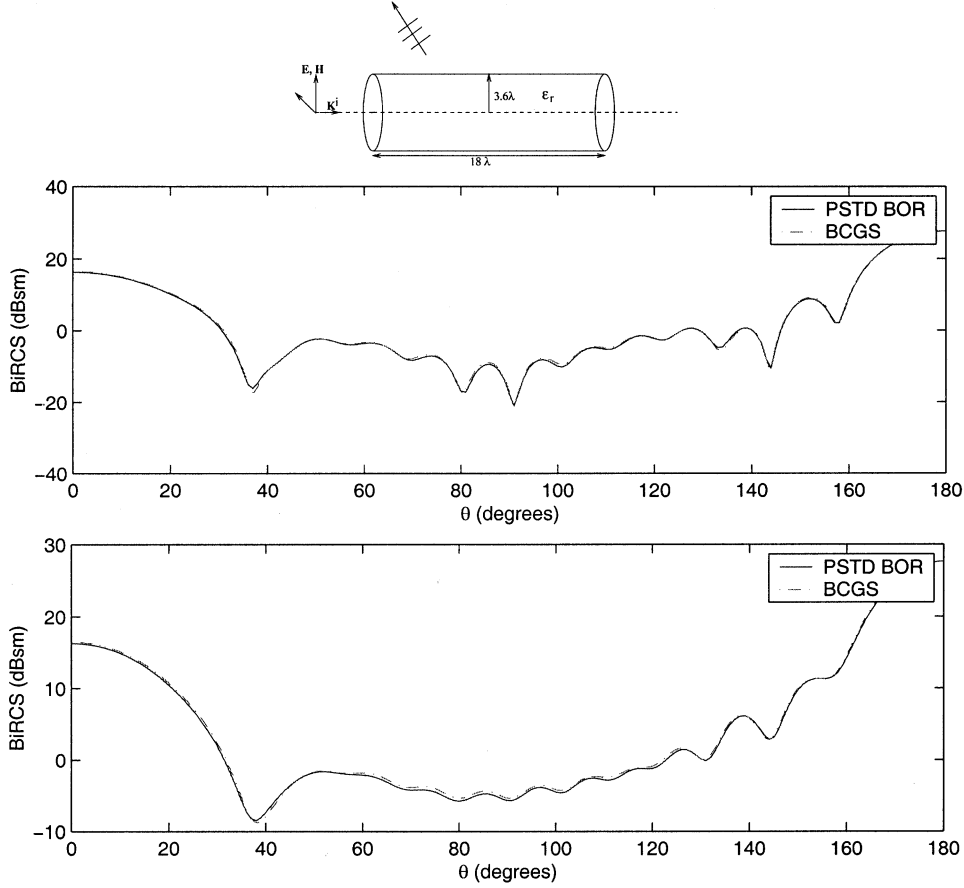


Fig. 5. RCS of a dielectric cylinder with $\epsilon_r = 12.96$, $\sigma = 0.3$ S/m. Frequency 1 GHz, cylinder length 1.5 m (18λ), and diameter 0.3 m (7.2λ).

$\mathcal{A}[\tilde{\mathbf{D}}]$ in terms of some basis functions (for example, the rooftop basis functions) $\mathbf{f}_{m,n,p}^{(\eta)}$

$$\begin{aligned}\tilde{\mathbf{D}}(\mathbf{r}) &= \sum_{\eta=x,y,z} d_{m,n,p}^{(\eta)} \mathbf{f}_{m,n,p}^{(\eta)}(\mathbf{r}), \\ \mathcal{A}[\tilde{\mathbf{D}}] &= \sum_{\eta=x,y,z} A_{m,n,p}^{(\eta)} \mathbf{f}_{m,n,p}^{(\eta)}(\mathbf{r}).\end{aligned}\quad (4)$$

It is remarked here that (2) is also expanded in terms of the basis functions. This second discretization step of expanding $\mathbf{A}$ in terms of the basis functions, although not necessary in formality, helps to weaken the singularity in (3).

Substituting (4) into (3) and (2) and testing (3) with the same basis functions (i.e., a Galerkin's procedure), we finally obtain

$$\mathcal{L}[\mathbf{d}] \equiv \sum_{\eta} \left\{ \mathbf{T}_1^{(\eta)} \mathbf{d}^{(\eta)} + \mathbf{T}_2^{(\eta)} \mathbf{A}^{(\eta)} \right\} = \mathbf{e} \quad (5)$$

where $\mathbf{d}$ is the vector containing unknown coefficients $d_{m,n,p}^{(\eta)}$, $\mathbf{e} = \{\mathbf{e}^{(x)}, \mathbf{e}^{(y)}, \mathbf{e}^{(z)}\}^T$, $\mathbf{e}^{(\eta)} = \langle \mathbf{f}^{(\eta)}, \mathbf{E}^{inc} \rangle$ and $\mathbf{T}_1^{(\eta)}$ and $\mathbf{T}_2^{(\eta)}$ are some simple “tri-tridiagonal” matrices, i.e., they are zero except only for elements $T_{m+k_1, n+k_2, p+k_3}^{(\eta)}$ where $k_i = -1, 0, 1$ ($i = 1, 2, 3$). It is important to note that only these nonzero elements are stored, thus the memory requirement is only $O(N)$. Furthermore, only the products of these nonzero elements with vectors $\mathbf{d}$ and $\mathbf{A}$ are calculated, thus the two matrix-vector products in (5) requires $O(N)$ CPU time. The explicit expressions of $\mathbf{T}_1^{(\eta)}$ and $\mathbf{T}_2^{(\eta)}$ are given in, [22] and [25]. The elements of

$\mathbf{A}^{(\eta)}$ are given by

$$\begin{aligned}A_{m,n,p}^{(\eta)} &= \sum_{m',n',p'} g_{m-m', n-n', p-p'} \chi_{m',n',p'}^{(\eta)} d_{m',n',p'}^{(\eta)} \\ &\equiv \left\{ g \otimes [\chi^{(\eta)} d^{(\eta)}] \right\}_{m,n,p}.\end{aligned}\quad (6)$$

Note that the 3-D linear discrete convolution in (6) denoted by $\otimes$ can be obtained by the FFT algorithm. Solving (5) for vector $\mathbf{d}$ gives the unknown electric flux density $\tilde{\mathbf{D}}$ inside V .

Iterative Solvers: Equations (5) and (6) can be written symbolically as $\mathcal{L}[\mathbf{d}] = \mathbf{e}$. This equation can be solved by various iterative solvers. Previously, for the weak-form formulation presented above, the CG method was used [22]. Recently, we utilized the BCG method [25]. The combination of the weak-form discretization and the BCG method significantly accelerates the convergence speed. More recently, we have applied the stabilized BCG method [30], [31] to further accelerate the convergence speed of the spectral method. As shown in the numerical results, up to a factor of 14 in convergence speedup can be achieved with the BCGS program compared to the CG method.

One important observation from the formulation is that the most expensive operation on left-hand side of (5) is the discrete convolution shown in (6). The products of “tri-tridiagonal” matrices $\mathbf{T}_1^{(\eta)}$ and $\mathbf{T}_2^{(\eta)}$ with vectors cost only $O(N)$ CPU time and $O(N)$ computer memory, where N is the total number of unknowns. Therefore, if the FFT algorithm is used to compute the discrete convolution (6) in an iterative method, the total CPU

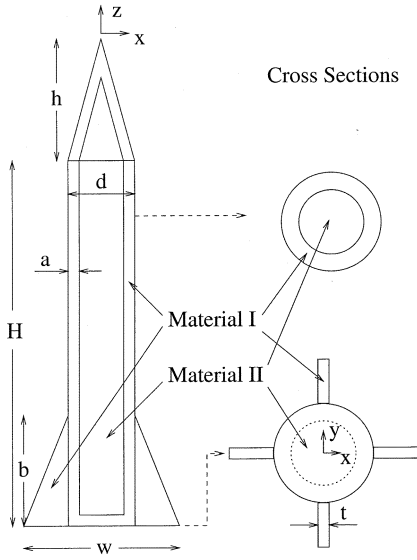


Fig. 6. Geometry of a missile-like object.

TABLE I
PARAMETERS FOR THE MISSILE-LIKE OBJECT IN FIG. 6.

Material I	Material II	a	b	d	h	H	t	w
$\epsilon_r = 2.25$	$\epsilon_r = 4$	10.5	50	60	100	665	6	94.5
$\sigma = 0.05$	$\sigma = 0.1$	cm	cm	cm	cm	cm	cm	cm

time is $O(KN \log N)$, where K is the iteration number and the total memory requirement is $O(N)$.

III. RESULTS AND DISCUSSIONS

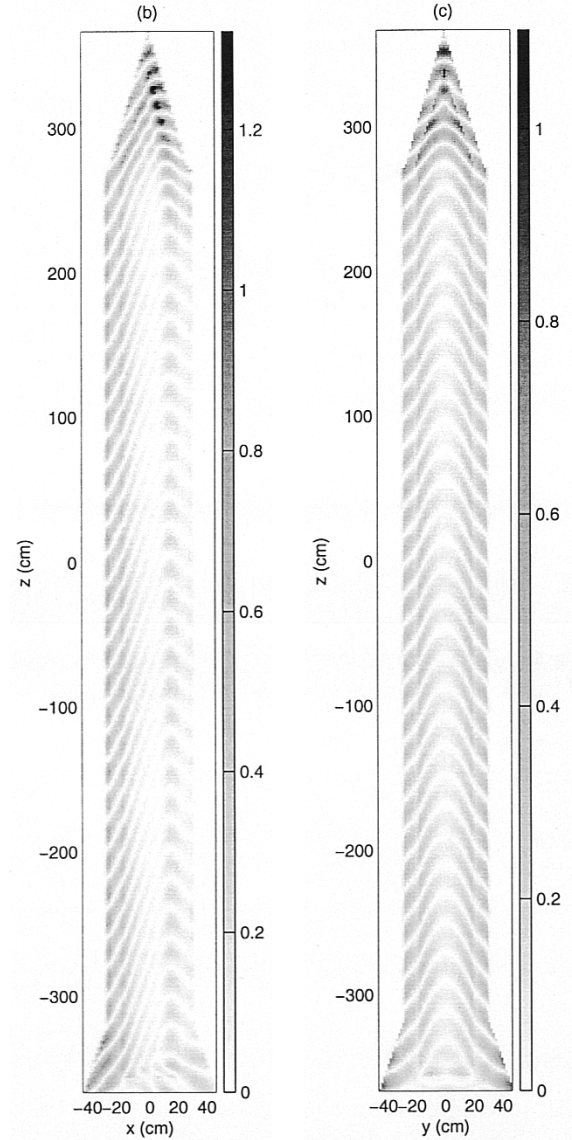
In the following numerical examples, we use the relative residual error criterion

$$\epsilon_k = \frac{\|\mathcal{L}\mathbf{d}_k - \mathbf{e}\|}{\|\mathbf{e}\|} \leq 10^{-3}$$

to terminate the iteration procedure. The incident plane wave is polarized in $\hat{x}$. Unless otherwise stated, the following results were obtained on an SGI Origin 2400 *single* 400-MHz MIPS R12000 processor. (The theoretical peak speed of the processor is 0.8 GFlops, but the actual speed is likely to be significantly less.) The total available memory on this single processor is about 5 GB.

We first compare the weak-form the BCG- and BCGS-FFT methods with the CG-FFT method for a two-layer sphere case that has been modeled in several papers [22], [24], [25]. The radii of the inner and outer spheres are $a_1 = 7.5$, $a_2 = 15$ cm, their dielectric constants are $\epsilon_{r1} = 72$, $\epsilon_{r2} = 7.5$, and their conductivities are $\sigma_1 = 0.9$, $\sigma_2 = 0.05$ S/m, respectively. The frequency is 100 MHz. The three methods converge to 0.1% error at significantly different rates, as shown in Fig. 2. The iteration numbers for CG, BCG, and BCGS are 360, 50, and 26, respectively. For this case, the BCG and BCGS methods are almost seven and 14 times faster than the CG method, respectively.

We then consider a moderate-size two-layer lossy dielectric sphere with the inner radius $a_1 = 1.0575$ m and outer radius $a_2 = 1.2075$ m. The relative permittivity and conductivity are $\epsilon_{r1} = 4$, $\sigma_1 = 0.1$ S/m, $\epsilon_{r2} = 2$, and $\sigma_2 = 0.05$ S/m for the inner and outer materials, respectively. A plane wave at 1 GHz is incident in the direction of $\theta = 0$ (i.e., along $+z$ direction).

Fig. 7. Electric current density on $x-z$ and $y-z$ cross sections.

At this frequency, the wavelength corresponding to the highest dielectric is $\lambda_1 = 15$ cm. Therefore, this represents a relatively large problem with an outer sphere diameter $2a_2 = 16.1\lambda_1$. With a discretization of ten cells per wavelength, this problem is discretized into a cube of $162 \times 162 \times 162$ cells. The total number of unknowns is about 12.75 million.

This problem was run on a SUN Ultra 60 Workstation. It requires 1.1 GB of memory. For the solution to converge to a 0.1% error, it takes 171 iterations using the BCG method and 20.27 min/iteration. Fig. 3 shows the bistatic RCS curves as a function of θ for $\phi = \pi/2$. Our numerical result has an excellent agreement with the Mie series.

Next, we model a large four-layer sphere with an incident plane wave at $f = 1$ GHz. The diameters of the layers are 1.515, 2.115, 2.415, and 2.865 m, respectively. The dielectric constant and conductivity of these layers are $\epsilon_r = 4, 2.56, 4, 2.25$, and $\sigma = 0.1, 0.07, 0.1, 0.02$ S/m, respectively. In terms of the minimum wavelength λ_1 (i.e., the wavelength for layers 1 and 3, which determines the highest spatial frequency), this 3-D problem has a volume of size $3648\lambda_1^3$. The problem is discretized by 21.23 million unknowns (roughly ten points per λ_1).

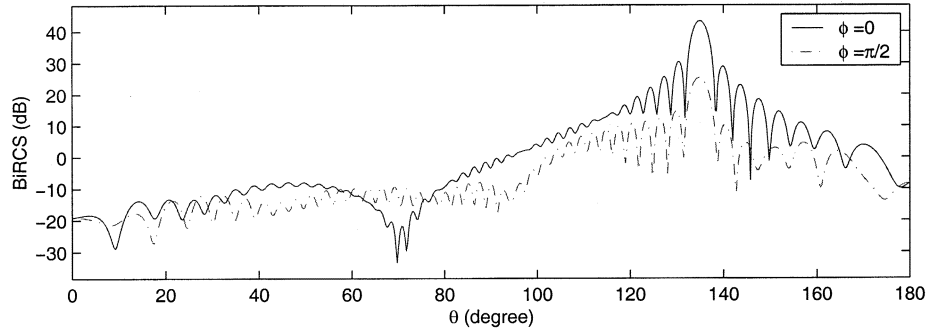


Fig. 8. RCS of a missile-like object in Fig. 6. The 1-GHz plane wave is incident along $\theta = \pi/4$ and $\phi = \pi/4$.

It was run on a single SGI processor. The RCS of this multi-layer sphere is shown in Fig. 4. Again, an excellent agreement has been achieved with the Mie series.

The previous two examples are for spheres, which have smooth interfaces between different regions of inhomogeneities. We also validate our numerical scheme with other numerical methods for objects that cannot be modeled by analytical methods. Fig. 5 shows the RCS of a large circular dielectric cylinder with a finite length. The cylinder is 1.5 m long, 0.3 m in diameter, and has a dielectric constant of $\epsilon_r = 12.96$ and conductivity $\sigma = 0.3$ S/m. The operating frequency is 1 GHz. This object is discretized with $N = 9.45$ million unknowns. The RCS computed by our BCG-FFT method is compared with an independent, pseudospectral time-domain (PSTD) result for bodies of revolution (BOR) [32]. Excellent agreement has been found between the two sets of results.

The last example is for a large penetrable missile-like object. The xz cross section is shown in the left part of Fig. 6. The yz cross section is the same as the xz cross section. The right half of Fig. 6 shows the xy cross section of the object. This missile-like object has a length of 7.65 m and a width of 0.945 m at the base (detail dimensions are listed in Table I). The object is discretized into $64 \times 64 \times 512$ cells, with a total of about 6.3 million unknowns. A plane wave at 1 GHz is incident at a direction of $\theta = 135^\circ$ and $\phi = 0$.

Fig. 7(a) and (b) display the induced electric current density along the $x - z$ and $y - z$ cross sections. Note that because of the symmetry in the geometry and the incident plane waves, the induced current is symmetric on $y - z$ cross section, but not on $x - z$ cross section. Fig. 8 shows the bistatic RCS curves as a function of θ for $\phi = 0$ and $\phi = \pi/2$, respectively. It should be noted that the staircasing error of this method will be more severe near the cone of the object, where the inner cylinder has a radius of only 1.3 wavelength in that material and has a sharp transition. Nevertheless, the current distribution seems to be reasonable.

The CPU times per iteration for the last three examples (Figs. 4, 5, and 8) are 64.68, 18.30, and 14.10 min, respectively. The convergence curves of these examples are shown in Fig. 9. Note that iterations for Fig. 4 are terminated at 0.5% as the result is already sufficiently accurate.

Discussions: The methods previously presented, as well as CG-FFT and related BCG-FFT methods, can be considered a class of spectral methods following the original k -space method designated by Bojarski [9]. These methods require only $O(N)$ computer memory and $O(KN \log N)$ CPU time, where N is the number of unknowns and K is the number of iterations.

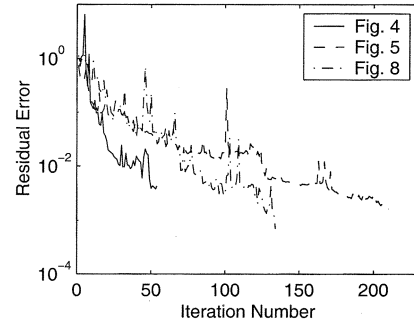


Fig. 9. Convergence curves for Figs. 4, 5, and 8. Iterations are terminated for Fig. 4 at 0.5% as the result is already sufficiently accurate.

We have applied these weak-form BCG-FFT and BCGS-FFT methods to solve large scale problems. However, we also note that several drawbacks of the methods require further improvement. First, this algorithm is most efficient when the dielectric object fills a solid rectangle prism. Otherwise, redundant unknowns are introduced in addition to the zero-padding (in the sphere examples, N is about 1.91 times that is needed). Second, there is a staircasing error associated with geometry discretization using rectangular prism cells. This second drawback can be improved upon by the related adaptive integral method [33].

IV. CONCLUSION

We present fast solution methods based on the weak-form discretization of the hypersingular electric field integral equation for the scattering problems from 3-D inhomogeneous objects. Our weak-form biconjugate-gradient and stabilized biconjugate-gradient FFT methods are shown to significantly outperform the CG-FFT method. Our methods are able to solve large-scale objects with 21 million unknowns even on a single workstation with 2-GB memory. Since the computer memory and CPU time requirements for these methods are $O(N)$ and $O(KN \log N)$, respectively, we estimate that with a 10-GB memory multiprocessor computer, we will be able to solve a problem with 100 million unknowns. In addition to the scattering problems, these methods have many other applications, including inverse scattering and microwave imaging.

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REFERENCES

- [1] J. H. Richmond, "Scattering by a dielectric cylinder of arbitrary cross section," *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 334–341, Mar. 1965.
- [2] —, "TE wave scattering by a dielectric cylinder of arbitrary cross-section shape," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 460–464, Mar. 1966.
- [3] V. Rokhlin, "Rapid solution of integral equations of classical potential theory," *J. Computat. Phys.*, vol. 60, pp. 187–207, 1983.
- [4] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas Propagat. Mag.*, vol. 35, pp. 7–12, 1993.
- [5] J. M. Song and W. C. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave Opt. Tech. Lett.*, vol. 10, pp. 15–19, 1995.
- [6] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1488–1493, Oct. 1997.
- [7] R. F. Harrington, *Field Computation by Moment Method*. Melbourne, FL: Krieger, 1968.
- [8] M. R. Hestenes and E. Stiefel, "Methods of conjugate gradient for solving linear systems," *J. Res. Nat. Bur. Stand.*, vol. 49, pp. 409–435, 1952.
- [9] N. N. Bojarski, "The k -space formulation of the scattering problem in the time domain," *J. Acoust. Soc. Amer.*, vol. 72, pp. 570–584, 1982.
- [10] T. K. Sarkar, E. Arvas, and S. M. Rao, "Application of fast Fourier transform and the conjugate gradient method for efficient solution of electromagnetic scattering from both electrically large and small conducting bodies," *Electromagn.*, vol. 5, pp. 99–122, 1985.
- [11] R. Kastner and R. Mittra, "Iterative analysis of finite-size planar frequency selective surfaces with rectangular patches or perforations," *IEEE Trans. Antennas Propagat.*, vol. 35, pp. 372–377, Apr. 1987.
- [12] T. A. Cwik and R. Mittra, "Scattering from a periodic array of free-standing arbitrarily shaped perfectly conducting or resistive patches," *IEEE Trans. Antennas Propagat.*, vol. 35, pp. 1226–1234, Nov. 1987.
- [13] T. J. Peters and J. L. Volakis, "Application of a conjugate gradient FFT method to scattering from thin planar material plates," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 518–526, Apr. 1988.
- [14] M. F. Catedra, J. G. Cuevas, and L. Nuno, "A scheme to analyze conducting plates of resonance size using the conjugate-gradient method and fast Fourier transform," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 1744–1752, Dec. 1988.
- [15] —, "A numerical scheme to obtain the RCS of three-dimensional bodies of resonance size using the conjugate-gradient method and fast Fourier transform," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 528–537, May 1989.
- [16] C. Y. Shen, K. J. Glover, M. I. Sancer, and A. D. Varvatsis, "The discrete Fourier transform method of solving differential-integral equations in scattering theory," *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1032–1041, Aug. 1989.
- [17] A. P. M. Zwamborn and P. M. van den Berg, "A weak form of the conjugate gradient FFT method for plate problems," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 224–228, Feb. 1991.
- [18] J. M. Jin and J. L. Volakis, "A biconjugate gradient FFT solution for scattering by planar plates," *Electromagn.*, vol. 12, no. 1, pp. 105–119, 1992.
- [19] Q. H. Liu and W. C. Chew, "A CG-FFHT method for the scattering solution of axisymmetric inhomogeneous media," *Microwave Opt. Tech. Lett.*, vol. 6, no. 2, pp. 101–104, 1993.
- [20] —, "Applications of the CG-FFHT method with an improved FHT algorithm," *Radio Sci.*, vol. 29, no. 4, pp. 1009–1022, 1994.
- [21] M. F. Catedra, R. P. Torres, J. Basterrechea, and E. Gago, *The CG-FFHT Method: Application of Signal Processing Techniques to Electromagnetics*. Boston, MA: Artech House, 1995.
- [22] P. Zwamborn and P. M. van den Berg, "The three-dimensional weak form of the conjugate gradient FFT method for solving scattering problems," *IEEE Trans. Microwave Theory Tech.*, vol. 9, pp. 1757–1766, Sept. 1992.
- [23] C. C. Su, "The three-dimensional algorithm of solving the electric field integral equation using face-centered node points, conjugate gradient method and FFT," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 510–515, Mar. 1993.
- [24] H. Gan and W. C. Chew, "A discrete BCG-FFHT algorithm for solving 3D inhomogeneous scatter problems," *J. Electromagn. Waves Applicat.*, vol. 9, no. 10, pp. 1339–1357, 1995.
- [25] Z. Q. Zhang and Q. H. Liu, "Three-dimensional weak-form conjugate and biconjugate-gradient FFT methods for volume integral equations," *Microwave Opt. Tech. Lett.*, vol. 29, no. 5, pp. 350–356, 2001.
- [26] C. Lanczos, "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," *J. Res. Nat. Bur. Stand.*, vol. 45, pp. 255–282, 1950.
- [27] —, "Solutions of systems of linear equations by minimized iterations," *J. Res. Nat. Bur. Stand.*, vol. 49, pp. 33–53, 1952.
- [28] T. K. Sarkar, "On the application of the generalized biconjugate gradient method," *J. Electromagn. Waves Applicat.*, vol. 1, no. 3, pp. 223–242, 1987.
- [29] C. F. Smith, A. F. Peterson, and R. Mittra, "The biconjugate gradient method for electromagnetic scattering," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 6, pp. 938–940, June 1990.
- [30] H. A. van der Vorst, "Bi-CGSTAB: A fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems," *SIAM J. Sci. Stat. Comput.*, vol. 13, no. 2, pp. 631–644, Mar. 1992.
- [31] Q. H. Liu, X. Xu, Z. Q. Zhang, and G. Zhao, "Strategy for modeling objects obstructed by foliage above a penetrable ground: Spectral and higher-order methods," in *2001 IEEE AP-S Intl. Symp. USNC/URSI National Radio Science Meet.*, Boston, MA, July 2001.
- [32] B. Yang and J. S. Hesthaven, "A pseudospectral method for time-domain computation of electromagnetic scattering by bodies of revolution," *IEEE Trans. Antennas Propagat.*, vol. 47, pp. 132–141, Jan. 1999.
- [33] E. Bleszynski, M. Bleszynski, and J. Jaroszewicz, "AIM: Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Sci.*, vol. 31, pp. 1225–1251, 1996.

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