

Lecture XV ADS - II - Competition

In this lecture, we try to replicate the ADS Momentum code. We do not consider the dielectric yet. We work with triangular patches only. We ignore the rectangular patches.

Part I MoM equations and the impedance matrix

1. MoM equations

According to Lecture#14, the MoM equations solved in ADS include:

1. Magnetic vector potential and electric scalar potential expressed through (surface) charges and currents:

$$\vec{A}(\vec{r}) = \mu_0 \int_{\mathcal{V}^3} G(\vec{r}, \vec{r}') \vec{J}(\vec{r}') d\vec{r}', \quad \varphi(\vec{r}) = \frac{1}{\epsilon_0} \int_{\mathcal{V}^3} G(\vec{r}, \vec{r}') \rho(\vec{r}') d\vec{r}', \quad j\omega\rho + \nabla \cdot \vec{J} = 0 \quad (1)$$

with the free-space Green's function:

$$G(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi\epsilon_0|\vec{r} - \vec{r}'|}$$

2. The boundary condition on the metal surface:

$$(j\omega\vec{A} + \nabla\varphi)_{\tan} = 0 \quad (2)$$

everywhere on the metal surface except for the port(s) .

Eqs. (1), (2) are the celebrated integral equations of electromagnetics solved by the Method of Moments (MoM). They allows us to find the unknown current and charge distributions on the metal surface. After that operation, the fields are found using Eqs. (9d,e) of the previous lecture

$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A} \quad (3)$$

$$\vec{E} = -\nabla\varphi - j\omega\vec{A} \quad (4)$$

Note: We emphasize again that the MoM method is the only method that works explicitly with charges and currents. The FDTD and FEM operate with fields. Therefore, the MoM must be *superior* to both FEM and FDTD when there exist small radiating objects - conductors (antennas, cellphones, rocket scatterers, etc.) separated by relatively large distances.

2. Rooftop basis functions

Below, we recall some properties of the most common rooftop basis functions on triangles (RWG basis functions). For any two triangular patches, t_n^+ and t_n^- , having areas A_n^+ and A_n^- , and sharing the common edge l_n , the basis function becomes

$$\vec{f}_n(\vec{r}) = \begin{cases} \frac{l_n}{2A_n^+} \vec{\rho}_n^+ & \vec{r} \text{ in } t_n^+ \\ \frac{l_n}{2A_n^-} \vec{\rho}_n^- & \vec{r} \text{ in } t_n^- \end{cases} \quad (5)$$

and (discuss the properties of the surface divergence, which is equal to 2 in this case)

$$\nabla \cdot \vec{f}_n(\vec{r}) = \begin{cases} \frac{l_n}{A_n^+} & \vec{r} \text{ in } t_n^+ \\ -\frac{l_n}{A_n^-} & \vec{r} \text{ in } t_n^- \end{cases} \quad (6)$$

where $\vec{\rho}_n^+ = \vec{r} - \vec{r}_n^+$ is the vector drawn from the free vertex of triangle t_n^+ to the observation point $\vec{r}$; $\vec{\rho}_n^- = \vec{r}_n^- - \vec{r}$ is the vector drawn from the observation point to the free vertex of triangle t_n^- . The following properties are of note:

1. The basis function is zero outside the two adjacent triangles t_n^+ and t_n^- ;
2. The RWG vector basis function is linear and has no flux (that is, has no normal component) through its boundary.
3. The component of the rooftop RWG vector basis function normal to the edge is always equal to one. The same is valid for other basis function modifications.

Note: For rooftop basis functions on rectangles, factor 2 in Eq. (5) would be missing.

Note: The most important property of the rooftop basis functions is the charge conservation law stated for every basis function by Eq. (6). Multiplying the charge density of the positive triangular patch by the area of this patch A_n^+ and the charge density of the negative triangular patch by the area of this patch A_n^- , we obtain exactly zero, i.e.

$$A_n^+ \times \frac{I_n}{A_n^+} + A_n^- \times -\frac{I_n}{A_n^-} = 0 \quad (7)$$

Thus, every basis function carries total charge equal to zero, so does any combination of such basis functions. Thus, the charge conservation law does not need to be introduced separately, it's already in place.

3. Scattering versus radiation. Inclusion of sources

Important note: Scattering (incident-wave driven solutions) or radiation (port-driven solutions) problems are ideologically identical – the only difference is in the following:

1. The incident field for a scattering problem is the radar field or another plane wave. This field is usually distributed over the entire target surface.
2. The “incident” field for a microstrip filter or an antenna is the applied electric field at the port (or in the feed). The incident field is concentrated only in the feed area and is zero everywhere else.

Therefore, only the scattering problem is considered here for simplicity. This has been the standard EM approach for a long time.

The total electric field is a combination of the incident field (labeled by superscript i) and the scattered field (labeled by superscript s), i.e.

$$\vec{E} = \vec{E}^i + \vec{E}^s \quad (8)$$

The incident electric field is either the incoming radar signal (scattering problem) or the excitation electric field at the port(s) (radiation problem). It is the scattered electric field $\vec{E}^s$ that is due to surface currents and free charges on the metal surface S - see Eq. (4) above.

$$\vec{E}^s = -\nabla\varphi - j\omega\vec{A} \quad (9)$$

The magnetic vector potential describes surface current radiation whereas the electric potential describes radiation of surface free charges. In the far field, both the φ -contribution and the $\vec{A}$ -contribution are equally important. On the metal surface S , the tangential component of the total electric field vanishes, $\vec{E}_{\text{tan}} = 0$, thus giving the electric field integral equation in its *standard form*:

$$E_{\text{tan}}^i = (j\omega\vec{A} + \nabla\varphi)_{\text{tan}} \quad \vec{r} \text{ on } S \quad (10)$$

4. Test functions

Assume that the test functions, $\vec{f}_m(\vec{r})$ $m = 1 \dots N$, cover the entire metal surface S and do not have a component normal to the surface. Multiplication of Eq. (10) by $\vec{f}_m(\vec{r})$ and integration over S gives N equations

$$\int_S \vec{f}_m \cdot \vec{E}^i ds = j\omega \int_S \vec{f}_m \cdot \vec{A} ds - \int_S (\nabla \cdot \vec{f}_m) \varphi ds \quad (11)$$

since, according to the divergence theorem and using standard vector identities,

$$\int_S \nabla\varphi \cdot \vec{f}_m ds = - \int_S \varphi (\nabla \cdot \vec{f}_m) ds \quad (12)$$

if $\vec{f}_m$ does not have a component perpendicular to the surface boundary or edge (if any).

5. Source functions

The surface current density, $\vec{J}$ is expanded into the basis functions (which usually coincide with the test functions) in the form

$$\vec{J}(\vec{r}) = \sum_{n=1}^N I_n \vec{f}_n(\vec{r}) \quad (13)$$

The magnetic vector potential has the form

$$\vec{A}(\vec{r}) = \mu_0 \int_S \vec{J}(\vec{r}') g(\vec{r}, \vec{r}') ds' \quad (14)$$

where μ_0 is the permeability in vacuum and $g(\vec{r}, \vec{r}') = (1/4\pi) \exp(-jkR)/R$, $R = |\vec{r} - \vec{r}'|$ is the free-space Green's function (time dependency $\exp(j\omega t)$ is assumed everywhere). In the expression for the Green's function $\vec{r}$ is the observation (test) point and $\vec{r}'$ is the

integration (source) point; both of them belong to the metal surface. After substitution of the expansion Eq. (13), the above equation becomes

$$\tilde{A}(\vec{r}) = \sum_{n=1}^N \left\{ \mu_0 \int_S \vec{f}_n(\vec{r}') g(\vec{r}, \vec{r}') ds' \right\} I_n \quad (15)$$

Similarly, the electric potential has the form

$$\varphi(\vec{r}) = \frac{1}{\epsilon_0} \int_S \rho(\vec{r}') g(\vec{r}, \vec{r}') ds', \quad j\omega\rho = -\nabla_S \cdot \vec{J} \quad (16)$$

It follows from equation (16) that ρ can be expressed in terms of the current density, through the surface divergence using the continuity equation. Hence the electric scalar potential reduces to

$$\varphi(\vec{r}) = \sum_{n=1}^N \left\{ \frac{1}{\epsilon_0} \frac{j}{\omega} \int_S (\nabla \cdot \vec{f}_n(\vec{r}')) g(\vec{r}, \vec{r}') ds' \right\} I_n \quad (17)$$

6. Moment equations

The moment equations are obtained if we substitute expansions (15) and (17) into the integral equation (11). In terms of symbolic notations,

$$\sum_{n=1}^N \hat{Z}_{mn} I_n = v_m, \quad m = 1, \dots, N \quad (18)$$

$$v_m = \int_S \vec{f}_m \cdot \vec{E}^i ds \quad (19)$$

where v_m are the “voltage” or excitation components for every test/basis function that have units of V·m. The integral expressions are the components of the impedance matrix $\hat{Z}$ of size $N \times N$, i.e.

$$\begin{aligned} Z_{mn} = & (j\omega\mu_0) \int_S \int_S \vec{f}_m(\vec{r}) \cdot \vec{f}_n(\vec{r}') g(\vec{r}, \vec{r}') ds' ds \\ & - \left(\frac{j}{\omega\epsilon_0} \right) \int_S \int_S (\nabla \cdot \vec{f}_m) (\nabla \cdot \vec{f}_n) g(\vec{r}, \vec{r}') ds' ds \end{aligned} \quad (20)$$

Note: The impedance matrix is symmetric for any set of basis functions (the test functions should be the same) when the corresponding surface integrals are calculated precisely. The components of the impedance matrix are the double surface integrals of the Green's function, and they mostly reflect the geometrical interaction between the elementary “dipole” RWG basis functions of the problem (discuss!).

In the matrix form, Eqs. (18-19) become

$$\hat{Z} \cdot \vec{I} = \vec{V} \quad (21)$$

Substitution of Eqs. (5), (6) into Eq. (20) gives the components of the impedance matrix in terms of RWG basis functions in the form

$$\begin{aligned} \iint_S \vec{f}_m \cdot \vec{f}_n g(\vec{r}, \vec{r}') ds' ds = & \\ & + \frac{l_m l_n}{4 A_m^+ A_n^+} \iint_{t_m^+ t_n^+} (\vec{\rho}_m^+ \cdot \vec{\rho}_n^+) g(\vec{r}, \vec{r}') ds' ds + \frac{l_m l_n}{4 A_m^+ A_n^-} \iint_{t_m^+ t_n^-} (\vec{\rho}_m^+ \cdot \vec{\rho}_n^-) g(\vec{r}, \vec{r}') ds' ds \\ & + \frac{l_m l_n}{4 A_m^- A_n^+} \iint_{t_m^- t_n^+} (\vec{\rho}_m^- \cdot \vec{\rho}_n^+) g(\vec{r}, \vec{r}') ds' ds + \frac{l_m l_n}{4 A_m^- A_n^-} \iint_{t_m^- t_n^-} (\vec{\rho}_m^- \cdot \vec{\rho}_n^-) g(\vec{r}, \vec{r}') ds' ds \end{aligned} \quad (22)$$

and

$$\begin{aligned} \iint_S (\nabla \cdot \vec{f}_m) (\nabla \cdot \vec{f}_n) g(\vec{r}, \vec{r}') ds' ds = & \\ & + \frac{l_m l_n}{A_m^+ A_n^+} \iint_{t_m^+ t_n^+} g(\vec{r}, \vec{r}') ds' ds - \frac{l_m l_n}{A_m^+ A_n^-} \iint_{t_m^+ t_n^-} g(\vec{r}, \vec{r}') ds' ds \\ & - \frac{l_m l_n}{A_m^- A_n^+} \iint_{t_m^- t_n^+} g(\vec{r}, \vec{r}') ds' ds + \frac{l_m l_n}{A_m^- A_n^-} \iint_{t_m^- t_n^-} g(\vec{r}, \vec{r}') ds' ds \end{aligned} \quad (23)$$

7. Integral calculations and impedance matrix filling

We have learnt that, for the low-order basis functions, the precise integration scheme does not add much to the accuracy of the numerical solution. Therefore, the rooftop basis functions, which in fact correspond to a piecewise constant (not linear) approximation on the metal surface (discuss separately), should be used in conjunction with the low-order order integration schemes, preferably, with the central-point integration.

The central-point integration allows us to reduce Eqs. (22), (23) to some expressions for the impedance matrix that could be easily vectorized in MATLAB or in another software. This is a huge advantage of the one-point approximation. When the approximation of all integrals at the center point is used, one has

$$\begin{aligned}
Z_{mn} = & \underbrace{\left(\frac{j\omega\mu_0 l_m l_n}{16\pi} \right) \left[(\vec{\rho}_m^+ \cdot \vec{\rho}_n^+) g_{mn}^{++} + (\vec{\rho}_m^+ \cdot \vec{\rho}_n^-) g_{mn}^{+-} + (\vec{\rho}_m^- \cdot \vec{\rho}_n^+) g_{mn}^{-+} + (\vec{\rho}_m^- \cdot \vec{\rho}_n^-) g_{mn}^{--} \right]}_{\text{current contribution}} \\
& + \underbrace{\left(\frac{j l_m l_n}{4\pi\omega\epsilon_0} \right) \left[g_{mn}^{++} + g_{mn}^{+-} + g_{mn}^{-+} + g_{mn}^{--} \right]}_{\text{charge contribution}}
\end{aligned} \tag{24}$$

$$g_{mn} = \frac{\exp(-jk|\vec{r}_m - \vec{r}_n|)}{|\vec{r}_m - \vec{r}_n|}, \quad \rho_m^\pm = \vec{r}_m - \vec{r}_{m0}^\pm$$

Here,

$\vec{r}_{m,n}$ are centers of the plus- and/or minus triangles, respectively;

$\rho_{m,n}^\pm$ are linear vectors drawn from the free vertex of the plus- and/or minus triangle to its center.

Eq. (24) could be written in the vector/matrix form

$$\begin{aligned}
\hat{Z} = & \underbrace{\left(\frac{j\omega\mu_0 l_m l_n}{16\pi} \right) \left[\hat{R}^{++} \cdot \hat{G}^{++} + \hat{R}^{+-} \cdot \hat{G}^{+-} + \hat{R}^{-+} \cdot \hat{G}^{-+} + \hat{R}^{--} \cdot \hat{G}^{--} \right]}_{\text{current contribution}} \\
& + \underbrace{\left(\frac{j l_m l_n}{4\pi\omega\epsilon_0} \right) \left[\hat{G}^{++} + \hat{G}^{+-} + \hat{G}^{-+} + \hat{G}^{--} \right]}_{\text{charge contribution}}
\end{aligned} \tag{25}$$

where all four square matrixes $\hat{G}^{++}, \hat{G}^{+-}, \hat{G}^{-+}, \hat{G}^{--}$ of the size $N \times N$ are obtained from the same matrix

$$\hat{G}, \quad g_{mn} = \frac{\exp(-jk|\vec{r}_m - \vec{r}_n|)}{|\vec{r}_m - \vec{r}_n|} \tag{26}$$

of the size $M \times M$ where M is the total number of triangular patches. These matrixes are obtained by substitution of the corresponding indexes into plus- and minus-triangles for every particular combination. Matrixes $\hat{R}^{++}, \hat{R}^{+-}, \hat{R}^{-+}, \hat{R}^{--}$ do not depend on the frequency and could be precalculated.

Part II Solution to MoM equations

1. Creation of basis functions

The following self-explanatory MATLAB code illustrates how to create the rooftop basis functions on triangles. A very similar treatment holds for rectangles and tetrahedra!

```
%-----
FacesTotal      = size(t,2);
NodesTotal      = size(P,2);
%-----

%-----
% Find base parameters for separate faces
CenterF = zeros(3, FacesTotal);
FCRho   = zeros(3, 3, FacesTotal);
for m = 1:FacesTotal
    Vertexes = P(:,t(1:3,m));
    r1       = Vertexes(:,1);
    r2       = Vertexes(:,2);
    r3       = Vertexes(:,3);
    C        = (r1+r2+r3)/3;
    AreaF(m) = norm(cross(r2-r1,r3-r1))/2;
    CenterF(:,m) = C;
    FCRho(:,1,m) = C- P(:,t(1,m));
    FCRho(:,2,m) = C- P(:,t(2,m));
    FCRho(:,3,m) = C- P(:,t(3,m));
end
%-----

%-----
% Create the rooftop basis functions:
% Find all edge elements "Edge" with at least two adjacent triangles
% (find all inner edges of the mesh). Also, for each triangle find free
% vertex(es) when this triangle is the PLUS triangle (array VerP) and
% when this triangle is the MINUS triangle (arrayVerM)
Edge = [];
n = 0;
for m = 1:FacesTotal
    N = t(1:3,m);
    for k = m+1:FacesTotal
        M = t(1:3,k);
        a = 1-all([N-M(1) N-M(2) N-M(3)]);
        b = 1-all([M-N(1) M-N(2) M-N(3)]);
        if(sum(a)==2) %triangles m and k have common edge
            n = n+1;
            Edge = [Edge M(find(a))];
            TriP(n) = m;
            TriM(n) = k;
            VerP(n) = find(~b);
            VerM(n) = find(~a);
        end
    end
end
EdgesTotal=length(Edge);
```

```

%-----
%   Prepare the symmetric distance matrix DIST (necessary for the symmetric
%   impedance matrix ZMM)
%   First row - distances from the center of face #1 to all other face
%   centers
%   Second row - distances from the center of face #2 to all other face
%   centers, etc.
DIST = zeros(FacesTotal, FacesTotal);
for m = 1:FacesTotal
    temp = CenterF - repmat(CenterF(:,m), 1, FacesTotal);
    DIST(m, :) = sqrt(dot(temp, temp));
end
%-----

%-----
%   Prepare the symmetric Rho matrices (necessary for the symmetric
%   impedance matrix ZMM)
%   First row - dot(Rho(:, m=1:EdgesTotal)*Rho(:, n=1))
%   Second row - dot(Rho(:, m=1:EdgesTotal)*Rho(:, n=2)), etc.
RhoPP = zeros(EdgesTotal, EdgesTotal);
RhoPM = zeros(EdgesTotal, EdgesTotal);
RhoMP = zeros(EdgesTotal, EdgesTotal);
RhoMM = zeros(EdgesTotal, EdgesTotal);
for m = 1:EdgesTotal
    RhoPP(m, :) = sum(FCRhoP.*repmat(FCRhoP(:,m), [1, EdgesTotal]));
    RhoPM(m, :) = sum(FCRhoP.*repmat(FCRhoM(:,m), [1, EdgesTotal]));
    RhoMP(m, :) = sum(FCRhoM.*repmat(FCRhoP(:,m), [1, EdgesTotal]));
    RhoMM(m, :) = sum(FCRhoM.*repmat(FCRhoM(:,m), [1, EdgesTotal]));
end
%-----
%-----
%   Self integrals IsIs(1/R) - IS
IS = zeros(1, FacesTotal);
for m = 1:FacesTotal
    IS(m) = self(P(:,t(1:3, m)));
end
%-----

%   Create structure geom
geom.FacesTotal = FacesTotal;           %   integer
geom.EdgesTotal = EdgesTotal;           %   integer

geom.t          = t;                     %   [4, FacesTotal]
geom.P          = P;                     %   [3, NodesTotal]
geom.AreaF      = AreaF;                 %   [1, FacesTotal]
geom.CenterF    = CenterF;               %   [3, FacesTotal]
geom.DIST       = DIST;                  %   [NodesTotal, NodesTotal]
geom.IS         = IS;                    %   [1, FacesTotal]

geom.EdgeLength = EdgeLength;            %   [1, EdgesTotal]
geom.Edge       = Edge;                  %   [2, EdgesTotal]
geom.TriP       = TriP;                  %   [1, EdgesTotal]
geom.TriM       = TriM;                  %   [1, EdgesTotal]
geom.VerP       = VerP;                  %   [1, EdgesTotal]
geom.VerM       = VerM;                  %   [1, EdgesTotal]
geom.RhoPP      = RhoPP;                 %   [EdgesTotal, EdgesTotal]

```

```

geom.RhoPM      = RhoPM;           % [EdgesTotal, EdgesTotal]
geom.RhoMP      = RhoMP;           % [EdgesTotal, EdgesTotal]
geom.RhoMM      = RhoMM;           % [EdgesTotal, EdgesTotal]

```

2. Filling out the impedance matrix

The following self-explanatory MATLAB code illustrates how to fill the impedance matrix. The script is fully vectorized. It's amazing to see how compact that script becomes compared, for example, to the FDTD method.

```

%-----
omega    = 2*pi*f;
k        = omega/const.c;
%-----

%-----
% Matrix of Green's functions
G = exp(-j*k*geom.DIST)./geom.DIST;
% Diagonal terms
for m = 1:geom.FacesTotal
    G(m, m) = geom.IS(m)-j*k;
end
%-----

%-----
% Impedance matrix filling
% Charge contribution
Z = +1/(4*pi*j*omega*const.epsilon)*(...
    G(geom.TriP, geom.TriP) - ...
    G(geom.TriP, geom.TriM) - ...
    G(geom.TriM, geom.TriP) + ...
    G(geom.TriM, geom.TriM));
% Current contribution
Z = Z + j*omega*const.mu/(16*pi)*(...
    G(geom.TriP, geom.TriP).*geom.RhoPP + ...
    G(geom.TriP, geom.TriM).*geom.RhoPM + ...
    G(geom.TriM, geom.TriP).*geom.RhoMP + ...
    G(geom.TriM, geom.TriM).*geom.RhoMM);
% Edge length multiplication
Z = Z.*kron(geom.EdgeLength, geom.EdgeLength');
%-----

```

3. Calculation of self-integrals

The self integrals of the impedance matrix do not allow for the central-point approximation and shall be calculated analytically. One has

$$\begin{aligned}
4\pi \int_{t_p} \int_{t_q} (\vec{\rho}_i \cdot \vec{\rho}'_j) g(\vec{r}, \vec{r}') ds' ds &= \int_{t_p} \int_{t_q} \frac{(\vec{\rho}_i \cdot \vec{\rho}'_j)}{|\vec{r} - \vec{r}'|} ds' ds + \int_{t_p} \int_{t_q} \frac{(\exp(-jk|\vec{r} - \vec{r}'|) - 1)(\vec{\rho}_i \cdot \vec{\rho}'_j)}{|\vec{r} - \vec{r}'|} ds' ds \\
&\approx (\vec{\rho}_i^C \cdot \vec{\rho}_j^C) \int_{t_p} \int_{t_q} \frac{1}{|\vec{r} - \vec{r}'|} ds' ds - jk(\vec{\rho}_i^C \cdot \vec{\rho}_j^C) A_p A_q
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
4\pi \int_{t_p} \int_{t_q} g(\vec{r}, \vec{r}') ds' ds &= \int_{t_p} \int_{t_q} \frac{1}{|\vec{r} - \vec{r}'|} ds' ds + \int_{t_p} \int_{t_q} \frac{(\exp(-jk|\vec{r} - \vec{r}'|) - 1)}{|\vec{r} - \vec{r}'|} ds' ds \\
&\approx \int_{t_p} \int_{t_q} \frac{1}{|\vec{r} - \vec{r}'|} ds' ds - jk A_p A_q
\end{aligned} \tag{28}$$

Thus, only one integral needs to be found, that is

$$\int_{t_p} \int_{t_q} \frac{1}{|\vec{r} - \vec{r}'|} ds' ds$$

This integral has been analytically calculated in Ref.

T. F. Eibert and V. Hansen, 1995, "On the calculation of potential integrals for linear source distributions on triangular domains," *IEEE Trans. Antennas Propagation*, vol. 43, no. 12, Dec. 1995, pp. 1499-1502.

Note: Demonstrate the advantages of the present approach compared to the old one - folder MATLAB_V1 to this lecture.

4. Ports/feeds

A. Delta-gap model

The concept of the port is shown in Fig. 1 that follows. The driving electric field (which is simultaneously the incident field, $\vec{E}^i$) is concentrated in the vicinity of the edge of a rooftop element. When the gap Δ is becoming small, the impressed electric field tends to infinity since its voltage (phasor) remains finite. Therefore,

$$\vec{E}^i \rightarrow V\delta(y)\mathbf{n}_y \quad \text{when } \Delta \rightarrow \infty \tag{29}$$

According to Eq. (19), for one basis function with the edge exactly passing through the port

$$v_m = \int_S \vec{f}_m \cdot \vec{E}^i ds = V \int_{t^+, t^-} \delta(y) \mathbf{n}_y \cdot \vec{f}_m ds = I_m V \quad (30)$$

Eq. (30) uses the fact that a component of the rooftop basis function normal to the edge is always equal to one.

All other components of the voltage excitation vector are equal to zero.

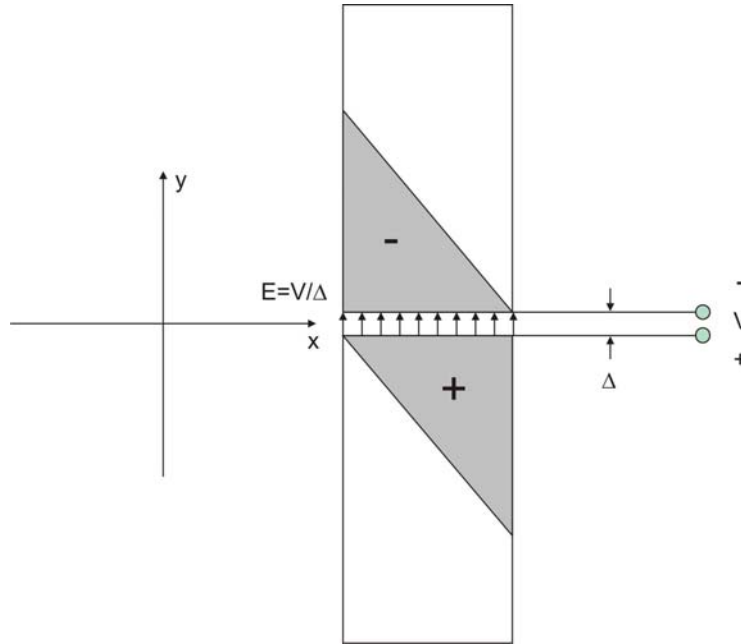


Fig. 1. Concept of the delta-gap feed/port in MoM.

After the MoM equations are solved, the total current through the port is found using the equality

$$I = I_m I_m \quad (31)$$

Knowing the port current we could find other port parameters, i.e.

```
Vg = 1.00; % source(generator) voltage fixed at 1 V
% The rest of code follows Ref. D. M. Pozar, Microwave Engineering,
% Wiley, 2005, 3rd edition, Chapter 4
% total current through excitation edge(s)
Current = sum(port_dir'.*I(Index1).*geom.EdgeLength(Index1)');
InputImpedance = Vg/Current;
resistance(m) = real(InputImpedance); % input resistance
reactance(m) = imag(InputImpedance); % input reactance
S11(m) = (InputImpedance - 50)/(InputImpedance + 50);
RL(m) = 20*log10(abs(S11(m)));
VSWR(m) = (1 + abs(S11(m)))/(1 - abs(S11(m)));
FeedPower(m) = 1/2*real(Current*conj(Vg));
```

B. Finite-gap model

The same idea applies, but an "artificial" rooftop element is built, which includes a separation rectangle of a finite width between two triangular metal patches - see Fig. 2. If the electric field applied at the port surface remains constant, Eqs. (30), (31) remain valid in this case too and do not have to be changed. The finite-gap port model has been introduced and used in Ref.

S. N. Makarov, S. D. Kulkarni, A. G. Marut, and L. C. Kempel, "Method of moments solution for a printed patch/slot antenna on a thin finite dielectric substrate using the volume integral equation," *IEEE Trans. Antennas and Propagation*, vol. **AP-54**, no. 4, pp. 1174-1184, April 2006.

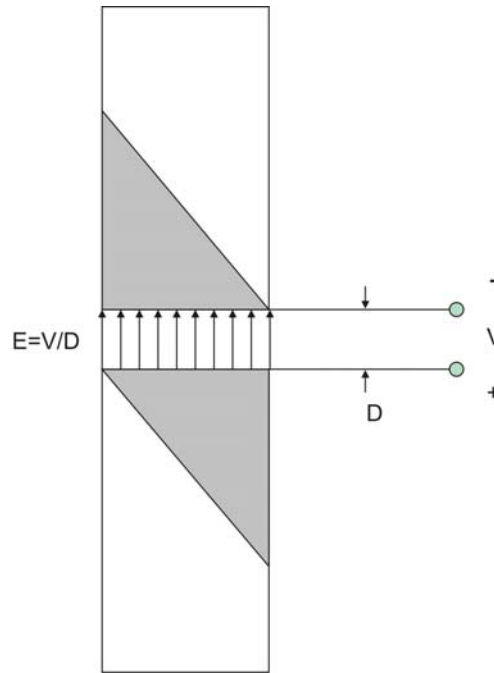


Fig. 2. Concept of the MoM port of finite width.

5. Port termination to lumped loads

For lumped loads (resistor, inductor, and capacitor), the diagonal elements of the impedance matrix should be modified by

$$\text{Resistance } R: \quad Z_{mm} \rightarrow Z_{mm} + l_m^2 R \quad (32a)$$

$$\text{Capacitance } C: \quad Z_{mm} \rightarrow Z_{mm} + l_m^2 \frac{1}{j\omega C} \quad (32b)$$

$$\text{Inductance } L: \quad Z_{mm} \rightarrow Z_{mm} + l_m^2 j\omega L \quad (32c)$$

This elegant result is obtained based on the fact that the voltage across the port now becomes

$$V = -Zl_m I_m \rightarrow v_m = -l_m V = -Zl_m^2 I_m \quad (33)$$

where Z is the load impedance. Next, the corresponding voltage drop is incorporated into moments equations and is moved to its left-hand side (to the impedance matrix) to obtain the modifications described by Eqs. (32). The sign minus in Eq. (33) is due to passive reference configuration that is opposite to the voltage source in Eqs. (29) and (30) when the current direction is that of Fig. 1 (from the plus triangle to the minus triangle).

Part III Test

The results for a two-port network (an antenna to antenna link, 220mm long blade dipoles) are tested against Ansoft HFSS - folder MATLAB_V2 to this lecture. Run the MATLAB script `main.m` to execute a frequency sweep. The impedance matrix is calculated at each frequency step.

Then, run the script `comparison.m` in order to compare the results with the corresponding Ansoft HFSS solution.

Discuss

1. Speed.
2. Accuracy.
3. Problems where MoM becomes superior. Problems where MoM will most likely fail.
4. Initiate a discussion which solver should be used for which problem (FDTD, FEM, MoM).
5. Present the plot (rather the sketch) of the speed.

The test results (impedance of the single dipole, return loss of the single dipole, and S21 (or insertion loss) for a two TX-RX port network) are given in the MATLAB scripts to this lecture (the script `comparison.m`). Fig. 3 that follows shows the S21 for two dipoles separated by 1.5m.

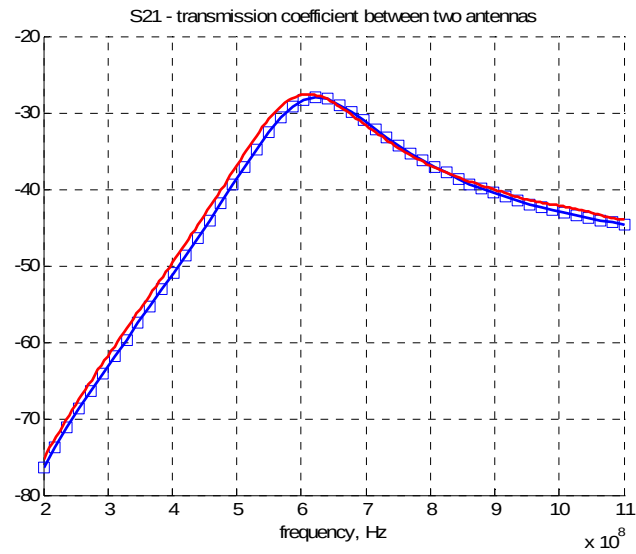


Fig. 3. S21 for two dipoles separated by 1.5m. Ansoft solution - solid curve; the MoM solution - squared curve.

HW#15 Scattering in MoM - is it simple?

1. An enemy target is a metal cylinder with the length of 2m and the diameter of 20 cm - see MATLAB folder **MATLAB_V3** to this lecture. The target is illuminated by a 75 MHz radar signal at normal incidence:

```
% Incident field (infinite plane wave)
f = 75e6;                               % scan frequency, Hz
k0 = 2*pi*f/const.c;                    % wave number
k = k0*[-1 0 0]';                       % wave vector

% Vertical polarization
% Ez = exp(-j*dot(k, r))
% Ex = 0
% Ey = 0
Pol = [0 0 1]';
```

2. An ECE 539 student is asked to compute and plot to scale the resulting magnitude of the electric current on the cylinder surface. Please do so by specifying the corresponding voltage excitation vector in the MATLAB script `excitation.m`. The rest of the code in the MATLAB folder **MATLAB_V3** to this lecture shall remain unchanged.
3. Please use a color printer for better current resolution.